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Comparison and Analysis of Strain Gauge Balance Calibration Matrix Mathematical Models

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Air Operations Division
Aeronautical and Maritime Research Laboratory

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ABSTRACT

The construction, comparison and analysis of three distinct strain gauge balance calibration matrix models with various orders of the calibration equations was conducted. The aims of the investigation were to identify the accuracy of the three different calibration matrix models and to analyse their behaviour with different data optimisation techniques. A computer program written in the C and X/Motif programming language has been developed to analyse the matrix models. Two different least squares methods and four optimisation techniques have been implemented within the software. The accuracy of each calibration model is evaluated using two statistical estimation methods. It was found that all three balance calibration models had similar behavior in terms of accuracy. The accuracy of the equation in estimating the loads experienced by the balance increases as the order of the calibration equation increases.

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Executive Summary

Wind tunnels are one of the primary sources of aerodynamic data for aerospace research. The Australian Defence Science and Technology Organisation (DSTO) operates two major wind tunnels at the Aeronautical and Maritime Research Laboratory (AMRL), one covers the low speed regime and the other covers the transonic speed regime. Results obtained from wind tunnel tests are used in many areas, such as aerodynamic research, aircraft design, and validation for computational fluid dynamics.

Achieving a high level of accuracy in wind tunnel test results is essential. Accuracy of the results depends on many factors, such as the data acquisition system and the force and moment measurement system. At AMRL, the primary force and moment measurement system is the multi-component, internally mounted, strain gauge balance.

A strain gauge balance must be calibrated before it can be used to measure forces and moments in the wind tunnel. The aim of the balance calibration is to obtain a set of calibration coefficients which enable the voltage output of the balance to be converted into the corresponding forces and moments. There are many ways to describe the relationship between the forces and moments, and voltage output for a particular balance. Due to the imperfection of balance design and manufacturing, and the combined loading condition during wind tunnel testing, second order and above calibration models are generally used to account for the interaction effect between different components of the balance. As the order of the calibration model increases, so too does the complexity of the mathematical expressions. For example, a general third order calibration model for a six component strain gauge balance has a total of 198 calibration coefficients.

Three different balance calibration models with different order calibration equations are investigated in this report. In addition, various calibration data optimisation techniques are applied to different calibration models. A computer program has been written to provide an efficient method for performing the comparison and analysis.

One of the main findings was that, of the 15 balance calibration equations used, the 2nd order 84 coefficient and 3rd order 96 coefficient equations provide a more accurate estimation than the lower order calibration equations for the relationship between voltage output and applied load.

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Sunny completed his Bachelor of Engineering in Aerospace Engeering and Bachelor of Business in Business Administration with First Class Honours at RMIT in 1998, and joined the Aeronautical and Maritime Research Laboratory at Fishermans Bend the following year. He is currently working in the Flight Mechanics area of Air Operation Division, and is specialising in experimental aerodynamics and wind tunnel test techniques.

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Yoel completed his Bachelor of Science in 1987 and his Bachelor of Engineering in Aeronautical Engineering in 1989, both at Sydney University, and joined the Aeronautical Research Laboratory at Melbourne the following year. He completed a Master of Business Administration in Technology Management in 1995 at Monash University. He has predominantly worked in Flight Mechanics and experimental aerodynamics in the Wind Tunnels. During this period he has accumulated experience in aerodynamics with the Jindivik, Tonic, PC-9, Mk82 store, Amphibious Transport (LPA) ship, and the Hydrographic Ship wind tunnel test programmes. He has also been responsible for the development of the wind tunnel data acquisition systems, and he has been involved with the Transonic Wind Tunnel Upgrade project.

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Nomenclature

Symbols which are not listed here are defined and described in the corresponding section of the report.

Symbol	*Description*
[C]	Calibration coefficient matrix
$\left[\hat{C}\right]$	Approximated calibration coefficient matrix
[H]	Applied load matrix
H_i	Applied load reading of i^{th} component
$H_{i,p}$	The estimated load/moment of the i^{th} component for the p^{th} calibration data point
$\hat{H}_{i,p}$	The measured load/moment of the i^{th} component for the p^{th} calibration data point
[R]	Voltage output matrix
R_i	Voltage output reading of i^{th} component
X	Axial force component
Y	Side force component
Z	Normal force component
l	Rolling moment component
m	Pitching moment component
n	Yawing moment component
se_i	Standard error of the calculated load/moment (dimensionless)
$\delta_{i,p}$	The difference between the estimated and measured load/moment
τ	Chauvenet's criterion
$se1_i$	Standard error (with dimensional unit)
μ	Mean

Subscript	
i = 1	Axial force component
i = 2	Side force component
i = 3	Normal force component
i = 4	Rolling moment component
i = 5	Pitching moment component
i = 6	Yawing moment component
p	Index for the calibration data.

1 Introduction

This work was carried out as part of the wind tunnel infrastructure program at the Aeronautical and Maritime Research Laboratory (AMRL). The aims of the investigation were to identify the accuracy of different strain gauge balance calibration matrix models and the characteristics of the model when used in combination with various data optimisation techniques. Based on the results from the investigation, an optimum calibration model is recommended for use in both the low speed and transonic wind tunnel facilities at AMRL. The software written for this investigation will be integrated into the existing data acquisition software to enable real-time conversion between strain gauge balance voltage output and the forces and moments experienced by the balance during the wind tunnel test.

Investigations into three distinct strain gauge balance calibration models with different order calibration equations were conducted. A computer program written in the C and the X/*Motif* computer language was developed for the analysis. Two least squares methods, four data optimisation techniques, and two statistical estimations have been implemented within the computer program. The computer program generates a calibration matrix by calculating the calibration coefficients. Using the calculated calibration matrix, a reverse calibration is applied by the program to obtain the estimated forces and moments. The accuracy of the calibration model is evaluated based on the ability of the calibration matrix to estimate the forces and moments compared with the measured values. The standard error of the data set is used as an indicator of accuracy for each calibration model in the computer program.

All three calibration models display very similar behaviour in terms of accuracy for different equation orders. As the order of the calibration equation increases, so too does the accuracy of the model. This is because the higher order models provide a more comprehensive description of the interaction effect between the balance's components. Both the 2nd order 84 coefficient calibration equation and the 3rd order 96 coefficient calibration equation achieved a significant reduction in standard error compared with the 2nd order 27 coefficient and the 3rd order 33 coefficient equations. As the equation order increases to the fourth order, the additional amount of interaction effect accounted for by the model compared with 3rd order equations is expected to be minimal. Hence, it is suggested that fourth order and above calibration models are not necessary.

The results indicate that high interactions between balance load components may lead to diverging results in the reverse calibration procedure. This is because the strain gauge balance voltage output for a particular loading condition may represent either positive or negative loads for a particular component.

In general, it is recommended that optimisation techniques, which require the elimination of calibration data points, should not be used. This is because, those data

points being eliminated actually represent the physical behaviour of the balance or the data acquisition system.

Due to the non-linear nature of calibration data, results showed that the calibration data, which covered only a narrow range of load, led to an increase in the calibration model's accuracy. This was because the regression model was more effective in modelling a narrower band of data with a higher degree of linearity.

Additionally, this work showed that the number of calibration data points has a significant impact on the values of the calculated calibration coefficients. If insufficient calibration data is provided, the least squares regression methods may fail to obtain a set of calibration coefficients. The calibration model may also fail to produce an accurate estimation of the measured forces and moments, or in some cases, it may produce diverging results in the reverse calibration procedure.

2 Balance Calibration Models

There are many ways in which a balance calibration model may be defined. This report concentrated on three different models, each model being distinct. The following balance calibration models have been investigated:

1. [R] = [C][H]
2. [H] = [C][R]
3. [H] = [C][R-H] (this is a general representation of this calibration model)

In order to compare the accuracy of these models extensively, the first, second and third order calibration equations of these models were investigated. (A complete listing of all equations for the three balance calibration models is given in Appendix A.)

The following table is a summary of the three balance calibration models and the corresponding orders of the calibration equations investigated in this report.

		Balance Calibration Model		
Order of Equation	***Number of Components***	*[R] = [C][H]*	*[H] = [C][R]*	*[H] = [C][R-H]*
1st	5	5 coefficients	5 coefficients	
	6	6 coefficients	6 coefficients	6 coefficients
2nd	5	20 coefficients	20 coefficients	
	6	27 coefficients	27 coefficients	27 coefficients
		84 coefficients	84 coefficients	84 coefficients
3rd	6	33 coefficients	33 coefficients	33 coefficients
		96 coefficients	96 coefficients	96 coefficients

Table 1. Summary of balance calibration models and orders of calibration equations

2.1 MODEL 1: [R] = [C][H]

The model currently used for strain gauge balances in the wind tunnels in the Air Operation Division (AOD) at AMRL is in the form of:

$$[R]=[C][H]$$

This equation describes the physical relationship between load and strain gauge output voltage, ie. the strain gauge voltage is a function of the applied load.

2.1.1 First order equations

Both six component and five component first order calibration equations are modelled by the software. At AMRL, six component strain gauge balances are primarily used to measure the aerodynamic forces and moments of aircraft and missile models, and five component strain gauge balances are used to measure aerodynamic forces and moments of stores released from aircraft in the transonic wind tunnel. The five component strain gauge balance does not measure the axial force component (X).

2.1.1.1 First order, six component equation: 6 coefficients

The first order equation consists of six terms for each component, and each equation corresponds to an individual component of the balance.

$$R_i = C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6$$

where, i = 1,...,6

2.1.1.2 First order, five component equation: 5 coefficients

The five component balance calibration equation consists of Y, Z, l, m and n components.

$$R_i = C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6$$

where, i = 2,...,6

2.1.2 Second order equations

Two second order equations were investigated, the 27 coefficient equation and the 84 coefficient equation for a 6 component balance. Additionally, a 20 coefficient equation for a 5 component balance was also investigated.

2.1.2.1 Second order, six component equation: 27 coefficients

The second order equation includes the addition of square and cross product terms but it does not include the cross product of absolute terms. This equation has a total of 27 calibration coefficients for each component.

$$R_i = C_{i,1}H_1 + C_{i,2}H_2 + \ldots\ldots + C_{i,6}H_6 + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2} + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6$$

where, i = 1,…,6

2.1.2.2 *Second order, five component equation: 20 coefficients*

The axial force component is not considered in the five component balance calibration equation. This equation includes both the square and cross product terms but there are no absolute cross product terms.

$$R_i = C_{i,2}H_2 + C_{i,3}H_3 + \ldots\ldots + C_{i,6}H_6 + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2} + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + \ldots\ldots + C_{i,56}H_5H_6$$

where, i = 2,…,6

2.1.2.3 *Second order, six component equation: 84 coefficients*

The second order 84 coefficient equation is based on the 27 coefficient second order equation and includes the cross product of absolute terms.

$$\begin{aligned} R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + \ldots\ldots + C_{i,6}H_6 \\ & + C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + \ldots\ldots + C_{i,|6|}|H_6| \\ & + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2} \\ & + C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + \ldots\ldots + C_{i,6|6|}H_6|H_6| \\ & + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6 \\ & + C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + \ldots\ldots + C_{i,|56|}|H_5H_6| \\ & + C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + \ldots\ldots + C_{i,5|6|}H_5|H_6| \\ & + C_{i,|1|2|}|H_1|H_2 + C_{i,|1|3|}|H_1|H_3 + \ldots\ldots + C_{i,|5|6|}|H_5|H_6 \end{aligned}$$

where, i = 1,…,6

2.1.3 Third order equations

Two different third order equations were investigated, the 33 coefficient equation and the 96 coefficient equation.

2.1.3.1 *Third order, six component equation: 33 coefficients*

The third order, six component 33 coefficient equation consists of single, square, cubic and cross product terms but there are no absolute cross product terms.

$$\begin{aligned} R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + \ldots\ldots + C_{i,6}H_6 \\ & + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2} \\ & + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6 \\ & + C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + \ldots\ldots + C_{i,666}H_6^{\,3} \end{aligned}$$

where, i = 1,…,6

2.1.3.2 *Third order, six component equation: 96 coefficients*

The 6 component third order 96 coefficient equation consists of single, square, cubic, cross product and absolute terms.

$$\begin{aligned} R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + \ldots\ldots + C_{i,6}H_6 \\ & + C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + \ldots\ldots + C_{i,|6|}|H_6| \\ & + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2} \\ & + C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + \ldots\ldots + C_{i,6|6|}H_6|H_6| \\ & + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6 \\ & + C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + \ldots\ldots + C_{i,|56|}|H_5H_6| \\ & + C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + \ldots\ldots + C_{i,5|6|}H_5|H_6| \\ & + C_{i,|1|2}|H_1|H_2 + C_{i,|1|3}|H_1|H_3 + \ldots\ldots + C_{i,|5|6}|H_5|H_6 \\ & + C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + \ldots\ldots + C_{i,666}H_6^{\,3} \\ & + C_{i,|111|}|H_1^{\,3}| + C_{i,|222|}|H_2^{\,3}| + \ldots\ldots + C_{i,|666|}|H_6^{\,3}| \end{aligned}$$

where, i = 1,…,6

2.2 MODEL 2: [H] = [C][R]

Instead of defining [H] as the independent variable in the calibration model, the following mathematical models treat [R] (the strain gauge balance output voltage) as the independent variable in the calibration equation. Therefore, this equation implies that the load is a function of the strain gauge output voltage.

$$[H]=[C][R]$$

2.2.1 First order equations

2.2.1.1 First order, six component equation: 6 coefficients

$$H_i = C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6$$

where, i=1,…,6

2.2.1.2 First order, five component equation: 5 coefficients

$$H_i = C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6$$

where, i=2,…,6

2.2.2 Second order equations

Two second order type equations were investigated, the 27 coefficient equation and the 84 coefficient equation for a 6 component balance. Additionally, a 20 coefficient equation for a 5 component balance was also investigated.

2.2.2.1 Second order, six component equation: 27 coefficients

$$\begin{aligned} H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + \ldots\ldots + C_{i,6}R_6 \\ & + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + \ldots\ldots + C_{i,66}R_6^{\,2} \\ & + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + \ldots\ldots + C_{i,56}R_5R_6 \end{aligned}$$

where, i=1,…,6

2.2.2.2 Second order, five component equation: 20 coefficients

$$\begin{aligned} H_i = {} & C_{i,2}R_2 + C_{i,3}R_3 + \ldots\ldots + C_{i,6}R_6 \\ & + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + \ldots\ldots + C_{i,66}R_6^{\,2} \\ & + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + \ldots\ldots + C_{i,56}R_5R_6 \end{aligned}$$

where, i=2,…6

2.2.2.3 *Second order, six component equation: 84 coefficients*

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + \ldots\ldots + C_{i,6}R_6 \\
& + C_{i,|1|}|R_1| + C_{i,|2|}|R_2| + \ldots\ldots + C_{i,|6|}|R_6| \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + \ldots\ldots + C_{i,66}R_6^{\,2} \\
& + C_{i,1|1|}R_1|R_1| + C_{i,2|2|}R_2|R_2| + \ldots\ldots + C_{i,6|6|}R_6|R_6| \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + \ldots\ldots + C_{i,56}R_5R_6 \\
& + C_{i,|12|}|R_1R_2| + C_{i,|13|}|R_1R_3| + \ldots\ldots + C_{i,|56|}|R_5R_6| \\
& + C_{i,1|2|}R_1|R_2| + C_{i,1|3|}R_1|R_3| + \ldots\ldots + C_{i,5|6|}R_5|R_6| \\
& + C_{i,|1|2|}|R_1|R_2 + C_{i,|1|3|}|R_1|R_3 + \ldots\ldots + C_{i,|5|6|}|R_5|R_6
\end{aligned}
$$

where, i=1,…,6

2.2.3 Third order equations

Two different third order equations were investigated, the 33 coefficient equation and the 96 coefficient equation.

2.2.3.1 *Third order, six component equation: 33 coefficients*

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + \ldots\ldots + C_{i,6}R_6 \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + \ldots\ldots + C_{i,66}R_6^{\,2} \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + \ldots\ldots + C_{i,56}R_5R_6 \\
& + C_{i,111}R_1^{\,3} + C_{i,222}R_2^{\,3} + \ldots\ldots + C_{i,666}R_6^{\,3}
\end{aligned}
$$

where, i=1,…,6

2.2.3.2 *Third order, six component equation: 96 coefficients*

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + \ldots\ldots + C_{i,6}R_6 \\
& + C_{i,|1|}|R_1| + C_{i,|2|}|R_2| + \ldots\ldots + C_{i,|6|}|R_6| \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + \ldots\ldots + C_{i,66}R_6^{\,2} \\
& + C_{i,|1|1}R_1|R_1| + C_{i,2|2|}R_2|R_2| + \ldots\ldots + C_{i,6|6|}R_6|R_6| \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + \ldots\ldots + C_{i,56}R_5R_6 \\
& + C_{i,|12|}|R_1R_2| + C_{i,|13|}|R_1R_3| + \ldots\ldots + C_{i,|56|}|R_5R_6| \\
& + C_{i,1|2|}R_1|R_2| + C_{i,1|3|}R_1|R_3| + \ldots\ldots + C_{i,5|6|}R_5|R_6| \\
& + C_{i,|1|2}|R_1|R_2 + C_{i,|1|3}|R_1|R_3 + \ldots\ldots + C_{i,|5|6}|R_5|R_6 \\
& + C_{i,111}R_1^{\,3} + C_{i,222}R_2^{\,3} + \ldots\ldots + C_{i,666}R_6^{\,3} \\
& + C_{i,|111|}|R_1^{\,3}| + C_{i,|222|}|R_2^{\,3}| + \ldots\ldots + C_{i,|666|}|R_6^{\,3}|
\end{aligned}
$$

where, i=1,…,6

2.3 MODEL 3: [H] = [C][R-H]

This calibration model assumes the loads measured by the balance are a function of both balance voltage output and applied load. (IAI Engineering Division, 1998)

Only six component equations are considered for this particular balance calibration model.

2.3.1 First order equation

$$H_i = C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6$$

where, i=1,…,6

Note: this first order equation is the same as the 6 component, first order equation of the [H]=[C][R], balance calibration model. (see Section 2.2.1.1)

2.3.2 Second order equations

Two second order type equations were investigated, the 27 coefficient equation and the 84 coefficient equation.

2.3.2.1 *Second order equation: 27 coefficients*

$$\begin{aligned} H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + \ldots\ldots + C_{i,6R}R_6 \\ & - \left(C_{i,11}{H_1}^2 + C_{i,22}{H_2}^2 + \ldots\ldots + C_{i,66}{H_6}^2\right) \\ & - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6\right) \end{aligned}$$

where, i=1,…,6

2.3.2.2 *Second order equation: 84 coefficients*

$$\begin{aligned} H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + \ldots\ldots + C_{i,6R}R_6 \\ & - \left(C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + \ldots\ldots + C_{i,|6|}|H_6|\right) \\ & - \left(C_{i,11}{H_1}^2 + C_{i,22}{H_2}^2 + \ldots\ldots + C_{i,66}{H_6}^2\right) \\ & - \left(C_{i,|1|1}H_1|H_1| + C_{i,2|2|}H_2|H_2| + \ldots\ldots + C_{i,6|6|}H_6|H_6|\right) \\ & - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6\right) \\ & - \left(C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + \ldots\ldots + C_{i,|56|}|H_5H_6|\right) \\ & - \left(C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + \ldots\ldots + C_{i,5|6|}H_5|H_6|\right) \\ & - \left(C_{i,|1|2}|H_1|H_2 + C_{i,|1|3}|H_1|H_3 + \ldots\ldots + C_{i,|5|6}|H_5|H_6\right) \end{aligned}$$

where, i=1,…,6

2.3.3 Third order equations

Two different third order equations were investigated, the 33 coefficient equation and the 96 coefficient equation.

2.3.3.1 *Third order equation: 33 coefficients*

$$\begin{aligned} H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + \ldots\ldots + C_{i,6R}R_6 \\ & - \left(C_{i,11}{H_1}^2 + C_{i,22}{H_2}^2 + \ldots\ldots + C_{i,66}{H_6}^2\right) \\ & - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6\right) \\ & - \left(C_{i,111}{H_1}^3 + C_{i,222}{H_2}^3 + \ldots\ldots + C_{i,666}{H_6}^3\right) \end{aligned}$$

where, i=1,…,6

2.3.3.2 *Third order equation: 96 coefficients*

$$
\begin{aligned}
H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + \ldots\ldots + C_{i,6R}R_6 \\
& - \left(C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + \ldots\ldots + C_{i,|6|}|H_6|\right) \\
& - \left(C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + \ldots\ldots + C_{i,66}H_6^{\,2}\right) \\
& - \left(C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + \ldots\ldots + C_{i,6|6|}H_6|H_6|\right) \\
& - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + \ldots\ldots + C_{i,56}H_5H_6\right) \\
& - \left(C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + \ldots\ldots + C_{i,|56|}|H_5H_6|\right) \\
& - \left(C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + \ldots\ldots + C_{i,5|6|}H_5|H_6|\right) \\
& - \left(C_{i,|1|2}|H_1|H_2 + C_{i,|1|3}|H_1|H_3 + \ldots\ldots + C_{i,|5|6}|H_5|H_6\right) \\
& - \left(C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + \ldots\ldots + C_{i,666}H_6^{\,3}\right) \\
& - \left(C_{i,|111|}\left|H_1^{\,3}\right| + C_{i,|222|}\left|H_2^{\,3}\right| + \ldots\ldots + C_{i,|666|}\left|H_{6.}^{\,3}\right|\right)
\end{aligned}
$$

where, i=1,…,6

3 Calculation of Least Squares Calibration Coefficients

Calibration coefficients, [C], are a set of constants, which are used to describe the loading characteristics of a strain gauge balance. To obtain an accurate description of the balance, an adequate number of data points are required. This number of data points is largely dependent on the balance calibration equipment available to an organisation. The distribution of applied loads should cover the maximum range of the balance and ideally it would be similar to the loads experienced by the balance during wind tunnel tests.

Using various types of regression models, a set of calibration coefficients may be obtained from a set of load data. In this report, two different types of least squares regression methods have been used to obtain a set of calibration coefficients. Least squares regression allows all six components of the balance to be loaded simultaneously. Hence, the interactions among various components are accounted for in the set of calibration coefficients. Additionally, the balance can be loaded in any particular order, hence a random and arguably more realistic loading matrix can be applied.

The two regression methods are described below for the 3rd order calibration equation, [R] = [C][H]. The same methodology applies to the other calibration models and equations.

3.1 Multivariable Regression Method

The mathematical expression of the 3rd order equation, [R]=[C][H] definition is given in Section 2.1.3.1.

The following assumptions have been made in this regression method:
1. random error is assumed to be zero;
2. the observed values of the independent variable (in this example, the value of [H]) are measured without error. All error is in [R].

Since the balance has six components (X, Y, Z, l, m, n), six expressions are used to represent each component of the balance. The entire set of p data points can be expressed using matrix notation, where p is the number of data points, as:

$$[R]=\begin{bmatrix} R_{1,1} & R_{2,1} & R_{3,1} & R_{4,1} & R_{5,1} & R_{6,1} \\ R_{1,2} & R_{2,2} & R_{3,2} & R_{4,2} & R_{5,2} & R_{6,2} \\ R_{1,3} & R_{2,3} & R_{3,3} & R_{4,3} & R_{5,3} & R_{6,3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{1,p} & R_{2,p} & R_{3,p} & R_{4,p} & R_{5,p} & R_{6,p} \end{bmatrix}$$

$$[H]=\begin{bmatrix} H_{1,1} & H_{2,1} & H_{3,1} & \cdots & H_{6,1}{}^{3} \\ H_{1,2} & H_{2,2} & H_{3,2} & \cdots & H_{6,2}{}^{3} \\ H_{1,3} & H_{2,3} & H_{3,3} & \cdots & H_{6,3}{}^{3} \\ \vdots & \vdots & \vdots & & \vdots \\ H_{1,p} & H_{2,p} & H_{3,p} & \cdots & H_{6,p}{}^{3} \end{bmatrix}$$

Note: the size of matrix [H] depends on the complexity of the calibration model. For example, in the 3rd order, 6 component, 96 coefficient equation, [H] will be a (p x 96) matrix.

Each component of the balance is represented by 33 'linear' and 'non-linear' calibration coefficients. The calibration coefficients calculated using this least squares method are only an approximation. This is because random errors are expected to exist among the data set due to various sources, such as electro-magnetic interference (EMI), random vibration on the test rig during the calibration process, and errors induced in the data acquisition and processing.

$$[\hat{C}]=\begin{bmatrix} C_{1,1} & C_{2,1} & C_{3,1} & C_{4,1} & C_{5,1} & C_{6,1} \\ C_{1,2} & C_{2,2} & C_{3,2} & C_{4,2} & C_{5,2} & C_{6,2} \\ C_{1,3} & C_{2,3} & C_{3,3} & C_{4,3} & C_{5,3} & C_{6,3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{1,666} & C_{2,666} & C_{3,666} & C_{4,666} & C_{5,666} & C_{6,666} \end{bmatrix}$$

The multivariable regression has the following expression (Sprent, 1969):

$$[\hat{C}]=\left([H]^T[H]\right)^{-1}[H]^T[R]$$

3.2 Ramaswamy Least Squares Method

The mathematical expression of the 3rd order equation, [R]=[C][H] is given in Section 2.1.3.1.

The Ramaswamy method (Lam, 1989) states that the calibration coefficients are found when the residual between the measured strain gauge output and that obtained from the calibration equation is a minimum. This can be expressed as:

$$e_i=\sum\left[C_{i,1}H_{1,p}+C_{i,2}H_{2,p}+C_{i,3}H_{3,p}+\cdots+C_{i,666}H_{6,p}{}^3-R_{i,p}\right]^2, \qquad \text{where } i=1,...,6.$$

For this particular 3rd order model, there are 33 coefficients for each component of the balance and p equations.

$$\begin{aligned}
&\sum\left[C_{i,1}H_{1,p}+C_{i,2}H_{2,p}+C_{i,3}H_{3,p}+\cdots+C_{i,666}H_{6,p}{}^3-R_{i,p}\right]\cdot H_{1,p}=0\\
&\sum\left[C_{i,1}H_{1,p}+C_{i,2}H_{2,p}+C_{i,3}H_{3,p}+\cdots+C_{i,666}H_{6,p}{}^3-R_{i,p}\right]\cdot H_{2,p}=0\\
&\sum\left[C_{i,1}H_{1,p}+C_{i,2}H_{2,p}+C_{i,3}H_{3,p}+\cdots+C_{i,666}H_{6,p}{}^3-R_{i,p}\right]\cdot H_{3,p}=0\\
&\qquad\qquad\vdots\\
&\sum\left[C_{i,1}H_{1,p}+C_{i,2}H_{2,p}+C_{i,3}H_{3,p}+\cdots+C_{i,666}H_{6,p}{}^3-R_{i,p}\right]\cdot H_{6,p}{}^3=0
\end{aligned}$$

By putting the equations above into matrix notation, the balance calibration coefficient matrix, [C] can then be calculated as follows:

$$[C]=[E]^{-1}[A]$$

where,

$$[E]=\begin{bmatrix} \sum H_{1,p}H_{1,p} & \sum H_{1,p}H_{2,p} & \sum H_{1,p}H_{3,p} & \cdots & \sum H_{1,p}H_{6,p}{}^{3} \\ \sum H_{2,p}H_{1,p} & \sum H_{2,p}H_{2,p} & \sum H_{2,p}H_{3,p} & \cdots & \sum H_{2,p}H_{6,p}{}^{3} \\ \sum H_{3,p}H_{1,p} & \sum H_{3,p}H_{2,p} & \sum H_{3,p}H_{3,p} & \cdots & \sum H_{3,p}H_{6,p}{}^{3} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum H_{6,p}{}^{3}H_{1,p} & \sum H_{6,p}{}^{3}H_{2,p} & \sum H_{6,p}{}^{3}H_{3,p} & \cdots & \sum H_{6,p}{}^{3}H_{6,p}{}^{3} \end{bmatrix}$$

$$[A]=\begin{bmatrix} \sum H_{1,p}R_{1,p} & \sum H_{1,p}R_{2,p} & \sum H_{1,p}R_{3,p} & \cdots & \sum H_{1,p}R_{6,p} \\ \sum H_{2,p}R_{1,p} & \sum H_{2,p}R_{2,p} & \sum H_{2,p}R_{3,p} & \cdots & \sum H_{2,p}R_{6,p} \\ \sum H_{3,p}R_{1,p} & \sum H_{3,p}R_{2,p} & \sum H_{3,p}R_{3,p} & \cdots & \sum H_{3,p}R_{6,p} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum H_{6,p}{}^{3}R_{1,p} & \sum H_{6,p}{}^{3}R_{2,p} & \sum H_{6,p}{}^{3}R_{3,p} & \cdots & \sum H_{6,p}{}^{3}R_{6,p} \end{bmatrix}$$

$$[C]=\begin{bmatrix} C_{1,1} & C_{2,1} & C_{3,1} & C_{4,1} & C_{5,1} & C_{6,1} \\ C_{1,2} & C_{2,2} & C_{3,2} & C_{4,2} & C_{5,2} & C_{6,2} \\ C_{1,3} & C_{2,3} & C_{3,3} & C_{4,3} & C_{5,3} & C_{6,3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{1,666} & C_{2,666} & C_{3,666} & C_{4,666} & C_{5,666} & C_{6,666} \end{bmatrix}$$

3.3 Five Component Strain Gauge Balance Calibration Equations

The multivariable and Ramaswamy least squares methods are also applicable to the calibration equations for a five component balance. However, due to the mathematical characteristics of both of these least squares methods which require matrix inversion, the axial component must be removed from both the applied loads matrix [H] and the voltage output matrix [R] before these least squares methods can be used.

4 Balance Reverse Calibration

The balance reverse calibration process uses the derived balance calibration coefficients, [C], to calculate the load experienced by the balance based on the strain gauge voltage output. Different balance calibration models require different reverse calibration procedures. The procedures applied to the three calibration models in Section 2 are given in the following four sections.

4.1 Model 1: [R] = [C][H]

Due to the nature of this particular model's equation, an iterative reverse calibration method is required (Galway, 1980; Cook, 1959). A brief summary of the method outlined by Cook (1959) is described below.

$$[F'] = [C1][F] + [C2][G]$$

where, [C1] is the linear calibration coefficient matrix;
[C2] is the non-linear calibration coefficient matrix;
[F'] is the apparent loads matrix;
[F] is the true loads matrix;
[G] is the true loads pairs matrix.

Hence, $[F] = [C1]^{-1}\{[F'] - [C2][G]\}$

$$[F] = [C1]^{-1}[F'] + [D][G] \qquad \text{where, } [D] = -\{[C1]^{-1}[C2]\}$$

In the first iteration, it is assumed there is no interaction between components of the balance, so that;

$$[F] \approx [F_1] = [C1]^{-1}[F'] \qquad \text{-- Step 1}$$

In the second iteration and onwards, the interactions between components are taken into consideration in the reverse calibration process. The true loads pairs matrix $[G_1]$ can be calculated using the $[F_1]$ matrix.

$$[F] \approx [F_2] = [F_1] + [D][G_1] \qquad \text{-- Step 2}$$

For further iterations, step two is repeated,

$$[F_3] = [F_1] + [D][G_2] \qquad \text{-- Step 3}$$

In general form, the reverse calibration process can be written as,

$$\boxed{[F_n] = [F_1] + [D][G_{n-1}]}$$

This iterative process is repeated until the values of [F] converge. In general, a converged solution can be obtained after between two and ten iterations, depending on the accuracy specified for the converged values.

4.2 Model 2: [H] = [C][R]

Unlike the other two calibration models, this particular model does not require an iterative reverse calibration procedure. Instead, the true load experienced by the balance can be calculated directly by multiplying the calibration coefficient matrix [C] by the strain gauge voltage output matrix [R].

This gives certain advantages over the iterative reverse calibration procedure. In the case of an iterative calibration procedure, the matrix which requires inversion will become larger, hence reducing the efficiency and accuracy of the results due to the inherited inaccuracy within the matrix inversion routine. If the matrix being inverted is a singular matrix, the reverse iterative calibration procedure cannot be achieved. Most importantly, a non-iterative reverse calibration procedure eliminates any possibilities of a diverged solution (see Section 7).

4.3 Model 3: [H] = [C][R-H]

This particular calibration model requires an iterative reverse calibration procedure because the true load [H] is a function of both voltage and true load [R-H]. The reverse calibration procedure is very similar to the method described in section 4.1.

A simplified version of this particular model is as follow:

$$[H]=[C1][R]-[C2][H'']$$

where, [H] is the true load matrix,
[R] is the voltage output matrix,
[H''] is the 2nd order or true load pairs matrix (obtained from [H]),
[C1] is the linear calibration coefficient matrix,
[C2] is the non-linear calibration coefficient matrix.

In the first iteration, it is assumed that there is no interaction between different components of the balance, hence;

$$[H_1]=[C1][R]$$

In the second iteration, the value of [H''] is obtained from $[H_1]$.

$$[H_2]=[C1][R]-[C2][H''_1]$$

In general form:

$$\boxed{[H_n]=[C1][R]-[C2][H''_{n-1}]}$$

The iterative process is repeated until the value of [H] converges. Depending on the level of accuracy, in general, [H] will converge after between two and ten iterations.

4.4 Five Component Balance Calibration Equations

To apply the reverse calibration method, listed in Section 4.1 and 4.2, to the five component 1st and 2nd order balance calibration equations of the [R]=[C][H] and [H]=[C][R] models, the five component matrix, used in the regression method, must be converted into a six component matrix by adding zeros and one to the axial force component.

The following example demonstrates a six component calibration coefficient matrix for a five component balance calibration equation.

Model: [R]=[C][H], 1st order five component calibration coefficient matrix.

	C1	C2	C3	C4	C5	C6
H_X	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
H_Y	0.00000	6.77281e−3	1.38834e−5	1.03006e−3	3.04363e−6	−2.83162e−4
H_Z	0.00000	7.09958e−5	3.24193e−3	−5.75365e−4	2.50119e−4	9.79163e−6
H_l	0.00000	−3.18513e−5	6.99459e−5	9.30475e−2	6.49542e−4	2.84688e−4
H_m	0.00000	−4.44862e−6	−1.64417e−5	1.03809e−3	3.97225e−2	8.74806e−4
H_n	0.00000	8.73207e−5	−6.00392e−6	−1.35077e−2	1.18139e−4	7.422104e−2

Five component calibration coefficient matrix obtained from least squares regression method.

Six component calibration coefficient matrix is formed by adding the axial force component

The above six component calibration coefficient matrix can now be used directly with the reverse calibration methods (see Section 4.1 and 4.2) to calculate the estimated forces and moments measured by a five component strain gauge balance.

This approach for transforming a five component to a six component calibration coefficients matrix may be applied to any order of calibration equation.

5 Statistical Analysis

The two statistical indicators used to analyse the accuracy of each calibration model are the standard error and the coefficient of multiple correlation.

5.1 Standard Error

Standard error provides an indication of the accuracy (the degree of dispersion) of the calculated loads and moments using the approximated calibration coefficient matrix, [C], as compared with the measured values. This parameter, given in *Equation 1*, can be used as a benchmark to compare the accuracy of various balance calibration models.

$$se1_i = \sqrt{\frac{\sum_{p=1}^{N}\left(\hat{R}_{i,p} - R_{i,p}\right)^2}{N - f}} \qquad \textit{Equation 1}$$

where, f is the number of degrees of freedom in the calibration equations. (*the number of degrees of freedom is equal to the number of calibration coefficients per component*),

N is the total number of data points used in the calibration data set,

$R_{i,p}$ is the measured p^{th} component of the strain gauge balance output,

$\hat{R}_{i,p}$ is the calculated (approximated) force or moment component,

$se1_i$ is the standard error with dimension [Newton or Newton.meter].

The standard error calculated using the above formula has a dimension of Newton or Newton·meter. To convert it into a dimensionless parameter, the standard error calculated from *Equation 1* is divided by the corresponding balance component's maximum design load. Hence,

$$se_i = \frac{\text{Standard Error}_i\,(se1_i)\,[\text{N or N}\cdot\text{m}]}{\text{Maximum Design Load Range}_i\,[\text{N or N}\cdot\text{m}]} \times 100\% \qquad \textit{Equation 2}$$

Similar to the definition of standard deviation, standard error can be used to describe the distribution of data points. For example, 1se represents 68% of the measured loads and moments values, 2se represents 95% of the total measured values and 3se represents 99.7% of the overall measured values.

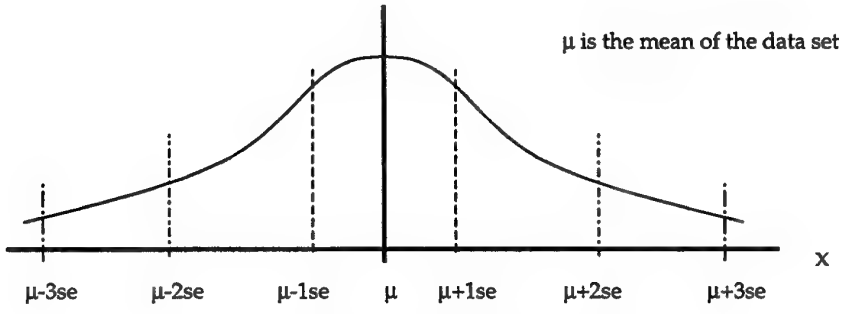


Figure 1. Standard error distribution

5.2 Coefficient of Multiple Correlation

The coefficient of multiple correlation, r_i, given in Equation 3, indicates how well the calibration equation describes the relationship between the outputs of the strain gauge balance and each component's loads. It also indicates the ability of the calibration equation to estimate the load measured by the strain gauge balance.

$$r_i = \sqrt{\frac{\sum_{p=1}^{N}\left(\hat{H}_{i,p} - \overline{H}_{i,p}\right)^2}{\sum_{p=1}^{N}\left(\hat{H}_{i,p} - \overline{H}_{i,p}\right)^2 + \sum_{p=1}^{N}\left(H_{i,p} - \hat{H}_{i,p}\right)^2}} \qquad \textit{Equation 3}$$

where, $\hat{H}_{i,p}$ is the estimated value of load/moment,

$\overline{H}_{i,p}$ is the mean of the component's load/moment for all the calibration data,

$H_{i,p}$ is the measured value of load/moment.

r_i is the coefficient of multiple correlation

The range of r_i is from 0 to 1. If $r_i = 1$, it means the correlation between the calibration model and the measured calibration data is perfect.

6 Data Optimisation

The aim of data optimisation is to improve the accuracy (reduce the scatter) of the estimated loads and moments as compared with the measured values. The results obtained from the optimisation process should have a high level of practicality. In other words, a zero or near zero standard error can be a meaningless representation of the accuracy of the calibration model and near zero standard errors could occur if only a small number of calibration data points have been sampled.

In this report, four different optimisation techniques have been used individually or in combination to investigate the overall effect of the results of various calibration models on the standard errors.

6.1 'Zero' Data Filter Optimisation

The function of the 'zero' data filter is to eliminate any values close to zero in the calibration data. This may be desirable because close to zero data points may be due to background noise instead of the actual loads or moments applied to the balance. This filter is applied to the calibration data before the calculation of the calibration coefficients.

6.2 Standard Error Optimisation

The standard error optimisation identifies potential "outliers", as shown in Figure 2, from the calibration data based on the standard error calculated for each individual component of the balance.

The standard error for each component of the balance must be converted from a percentage to the corresponding dimensional unit, b_i.

$$b_i = \frac{se_i \times \text{maximum design load}_i}{100}$$

$$\delta_{i,p} = \left| H_{i,p} - \hat{H}_{i,p} \right|$$

where, b_i is the outlier tolerance,
$\delta_{i,p}$ is the difference between the estimated and measured load / moment,
$H_{i,p}$ is the estimated load / moment,
$\hat{H}_{i,p}$ is the measured load / moment.

The condition selected for an outlier is:

$$\boxed{\delta_{i,p} \geq b_i}$$

In the computer program (see Section 8), if an outlier is found in a line of data (ie. one calibration point), the entire data line is eliminated from the calibration coefficient calculation. The definition of outliers in a graphical representation is shown in Figure 2.

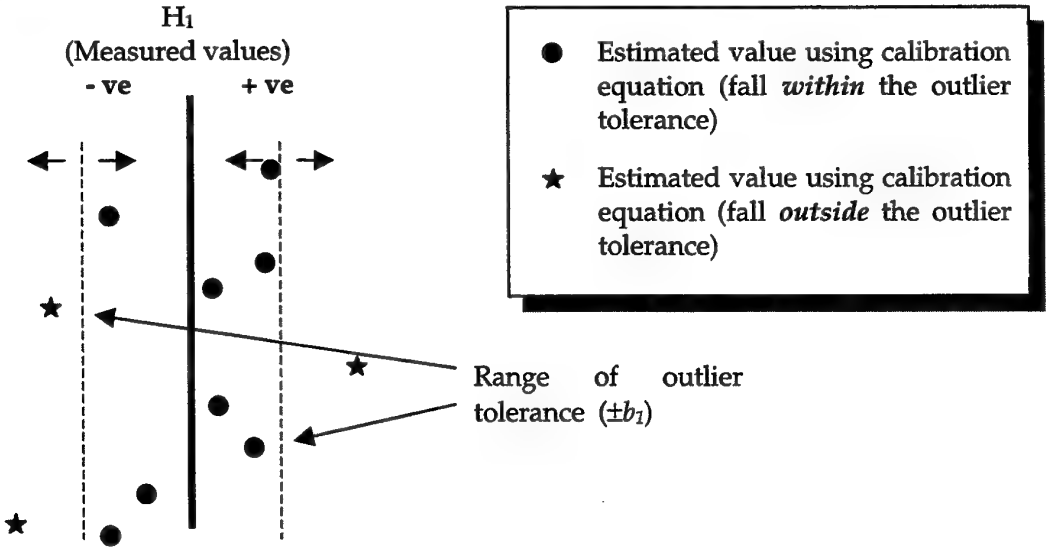


Figure 2. Standard error optimisation – definition of outlier

6.3 Chauvenet's Criterion Optimisation

After the calibration coefficients are calculated, an outlier elimination process based on Chauvenet's Criterion can be applied to the calibration data set to reduce the standard error of the results. Chauvenet's Criterion detects and eliminates potential outliers calculated by the calibration coefficient matrix [C] through the reverse calibration process. (AIAA, 1995)

The Chauvenet's Criterion defines potential outliers using the following relations,

$$\delta_{i,p} = \left| H_{i,p} - \hat{H}_{i,p} \right|$$

The condition selected for an outlier is:

$$\boxed{\delta_{i,p} \geq \tau \cdot se}$$

where, τ is the Chauvenet's Criterion, which can be calculated by the expression:

$$\tau = \sum_{i=0}^{5} A_i \left[\ln(N)\right]^i$$

where, $A_0 = 0.720185$ $A_1 = 0.674947$
$A_2 = -0.0771831$ $A_3 = 0.00733435$
$A_4 = -0.00040635$ $A_5 = 0.00000916028$

The above expression is only a curve-fit equation for τ using Chauvenet's Criterion for N<833,333, where N is the total number of data points used in the data set.

6.4 Optimised Calibration Matrix With Non-Optimised Calibration Data

In this optimisation technique, a set of optimised calibration coefficients are obtained by either the standard error optimisation (see Section 6.2) or the Chauvenet's Criterion optimisation (see Section 6.3). A reverse calibration process is then carried out on the optimised calibration matrix with the original calibration data set. The aim of this optimisation technique is to investigate the relationship between the optimised calibration matrix and the entire set of original (non-optimised) calibration data. In effect, this shows how well the optimised calibration matrix represents the original full data set.

7 Balance Calibration Models Analysis

The different balance calibration equations have been investigated using the computer program – *CALIB* (refer to Section 8), which has been developed by the author. The aim of the investigation was to identify the accuracy of each individual model based on its standard errors (see Section 5.1).

In the ideal situation, where the strain gauge balance has no interaction between different components, a simple first order balance calibration mathematical equation can accurately convert the balance voltage output to its corresponding load.

In reality, due to the imperfection in balance design, manufacture and deformation under load, a certain degree of interaction between components of the balance always exists. Hence, a higher order balance calibration mathematical model is required to account for the component interaction. There are many ways in which a balance calibration model may be defined. This report concentrated on three different models, each model being distinct (see Section 2).

Two sets of calibration data have been used for the analysis. One set consisted of 1886 data points obtained for the six component Aerotech strain gauge balance, shown in Figure 3, now being used at AMRL. Another set of calibration data consisted of 329 data points for the Collins six component strain gauge balance, shown in Figure 4. The aim of using two sets of data of significantly different size is to investigate the effect of the size of the calibration data on a particular calibration model in terms of accuracy. However, care must be exercised in drawing definitive conclusions about the models from only two data sets. A complete set of graphs showing the standard errors for both data sets, for each of the models, is provided in Appendix D.

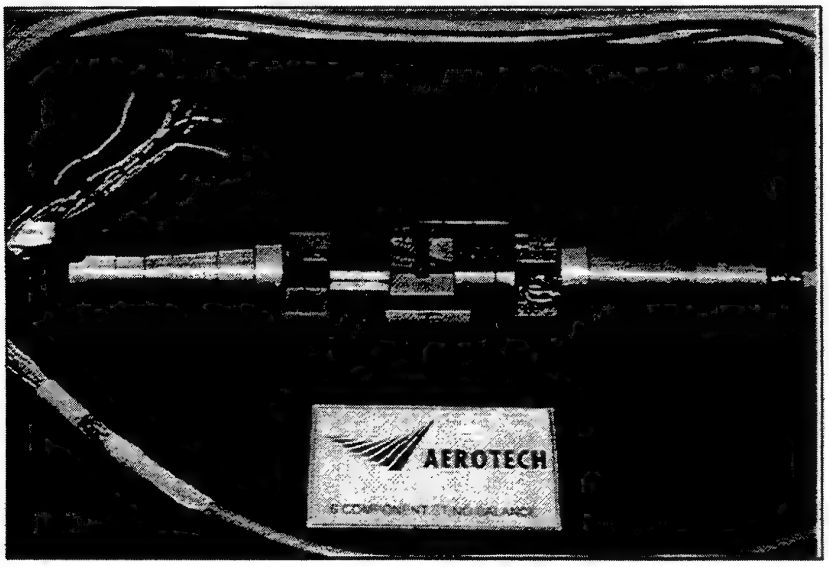


Figure 3. Transonic wind tunnel six component strain gauge balance designed by Aerotech

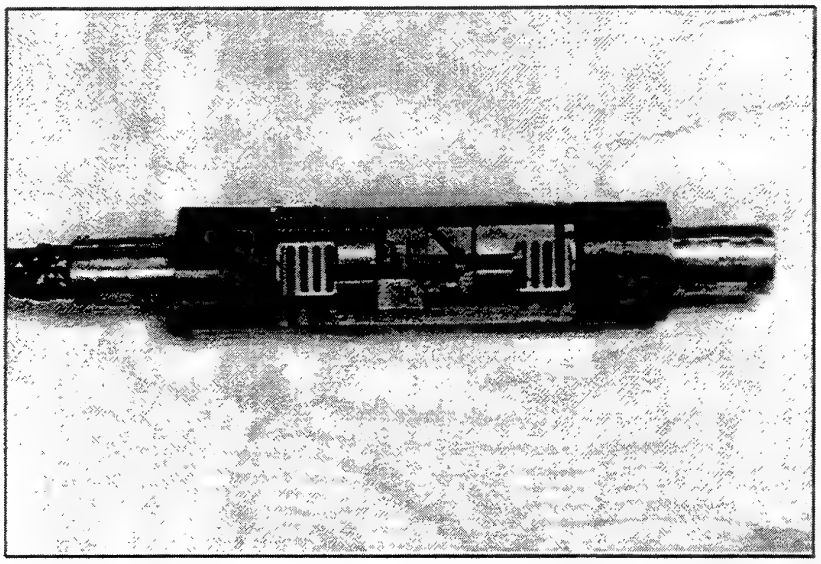


Figure 4. Collins six component strain gauge balance used in the AMRL low speed wind tunnel

7.1 Number of Calibration Data Points

An adequate number of calibration data points are required in order to calculate calibration coefficients using least squares regression methods. For example, computation of the [H]=[C][R-H] 2nd order 84 coefficient equation with one standard error optimisation failed during the reverse calibration process for the Aerotech balance (1886 data points), because some of the estimated values calculated using the optimised calibration matrix failed to converge after 6 iterations. This behaviour was caused by the elimination of a large number of the calibration data points (83% of the total number of data points) after the one standard error optimisation process (see Section 6.2). This significant reduction in the number of calibration data points caused the least squares regression to fail to generate an accurate description of the balance behaviour. However, for the 2se and 3se optimisation case, only 21.7% and 5.2% of the data have been rejected and the solution converges. (see Figure 12 in Appendix D.3)

All three models have similar standard error values for the same corresponding order of the calibration equation. From the analysis, there is no firm indication that one particular model is superior to the others in terms of accuracy (see Appendix D). With the 1886 data set, using a higher order equation, such as the 2nd order 84 coefficient equation and the 3rd order 96 coefficient equation, the [H]=[C][R-H] and [R]=[C][H] models have a slightly lower standard error ($\approx 3.64\times10^{-4}\%$) compared with the [H]=[C][R] model.

In terms of the reverse calibration process, model [H]=[C][R] has the advantage of using a simple non-iterative process, which eliminates the possibility of diverging reverse calibration results. In the other two models, if the interaction between different

components of the balance is relatively large, there is a possibility that an iterative reverse calibration process may fail to converge. This may occur when a particular voltage output from the balance, has both a positive and a negative force or moment for a particular component.

7.2 Effect of Balance Calibration Equation Order

As the order of the balance calibration equation increases, the accuracy of the model also increases. This is represented by a reduction in standard error for each balance component. For example, the standard errors for the [R]=[C][H] model are shown in Table 2 and Figure 5. Since each component of the balance has a different degree of interaction, the standard error also varies between components. A second order definition with 27 coefficients has a significant reduction in standard error, especially in components with high interaction behaviour. This is shown by a reduction of 0.385% (0.60054% to 0.21562%) in the standard error for the rolling moment component of the Aerotech balance, as compared with the first order 6 coefficient equation. A further 0.075% (0.21562% to 0.14070%) reduction in standard error for the rolling moment component is achieved by using the second order 84 coefficient equation, and 0.079% (0.21562% to 0.13685%) by using the third order 96 coefficient equation.

	Standard Error [%]						
	HX	**HY**	**HZ**	**Hl**	**Hm**	**Hn**	**Average**
6 coeff.	0.43587	0.16990	0.13214	0.60054	0.07482	0.14332	0.25943
27 coeff.	0.05858	0.11579	0.08537	0.21562	0.06154	0.09877	0.10595
33 coeff.	0.05654	0.11171	0.08477	0.21083	0.05641	0.09307	0.10222
84 coeff.	0.05183	0.08053	0.07138	0.14070	0.04497	0.05999	0.07490
96 coeff.	0.05069	0.07810	0.06969	0.13685	0.04379	0.05900	0.07302

Table 2. Standard error for the [R]=[C][H] balance calibration model with 1886 data points

The standard error reduces as the order of the calibration equation increases. As shown in Table 2, there is only an average of 0.002% (0.07490% to 0.07302%) improvement in standard error between the second order 84 coefficient and the third order 96 coefficient equation. Although the average improvement in accuracy is low, components with a relatively high degree of interaction, such as the rolling moment component, Hl, achieved a more significant reduction in standard error of 0.004% (0.14070% to 0.13685%) for the 3rd order, 96 coefficient equation of the [R]=[C][H] model compared with the 2nd order, 84 coefficient equation of the same model. Therefore, the use of the 3rd order 96 coefficient calibration equation can further improve the accuracy in estimating the load experienced by the balance, in particular for components with a high degree of interaction.

Similar trends for the other models are given in Table 3 and Table 4.

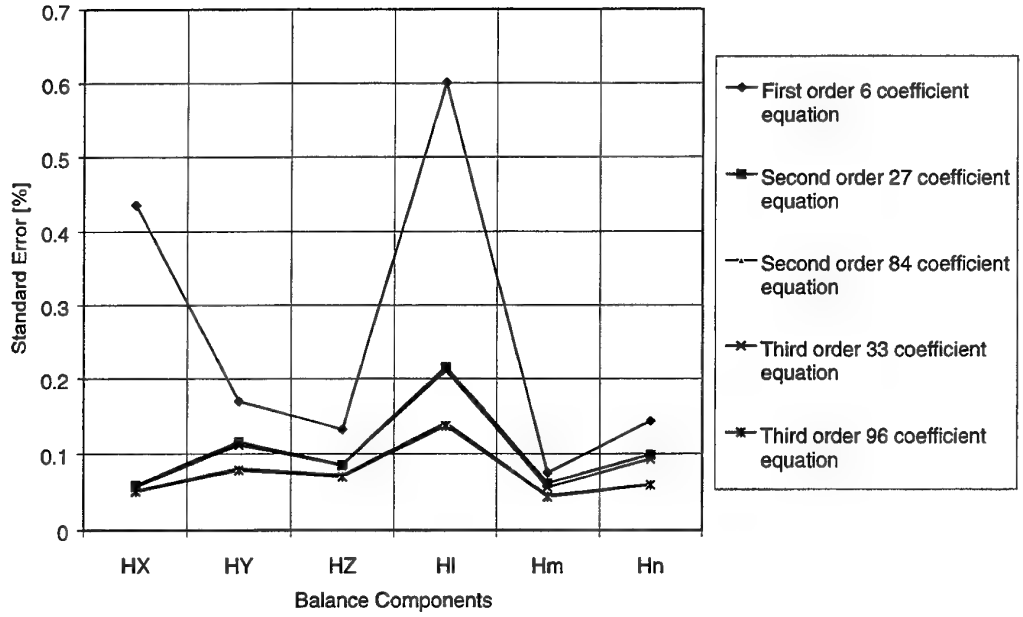


Figure 5. Balance Calibration Model: [R]=[C][H] with 1886 data points

	Standard Error [%]						
	HX	HY	HZ	Hl	Hm	Hn	Average
6 coeff.	0.43583	0.16990	0.13213	0.60045	0.07482	0.14332	0.25941
27 coeff.	0.05860	0.11663	0.08537	0.21537	0.06149	0.09848	0.10599
33 coeff.	0.05665	0.11216	0.08473	0.21040	0.05645	0.09321	0.10227
84 coeff.	0.05187	0.08055	0.07129	0.14107	0.04508	0.06077	0.07511
96 coeff.	0.05079	0.07794	0.06952	0.13771	0.04389	0.05986	0.07329

Table 3. Standard error for the [H]=[C][R] balance calibration model with 1886 data points

	Standard Error [%]						
	HX	HY	HZ	Hl	Hm	Hn	Average
6 coeff.	0.43583	0.16990	0.13213	0.60045	0.07482	0.14332	0.25941
27 coeff.	0.05858	0.11579	0.08537	0.21562	0.06154	0.09877	0.10595
33 coeff.	0.05653	0.11171	0.08477	0.21083	0.05641	0.09307	0.10222
84 coeff.	0.05182	0.08045	0.07142	0.14086	0.04499	0.05990	0.07491
96 coeff.	0.05058	0.07812	0.06966	0.13702	0.04385	0.05893	0.07303

Table 4. Standard error for the [H]=[C][R-H] balance calibration model with 1886 data points

7.3 Balance Calibration Coefficients Calculation

Both the multivariable regression and Ramaswamy least squares method produced the same calibration coefficients. Both methods require matrix inversion at some stage of the process, therefore, these methods will fail if the matrix being inverted is a singular matrix. Out of the two regression models, the multivariable regression is easier to program than the Ramaswamy least squares method.

7.4 Data Optimisation

7.4.1 Standard Error Optimisation

Using the 1886 data set, a reduction in standard errors is achieved by applying the standard error optimisation process (see Section 6.2). In all three models, a significant reduction in standard error is achieved by applying 1se optimisation. The increase in the model accuracy is due to the large amount of data being excluded from the coefficient calculation process. For the 1886 data set, the standard error optimisation process leads to an 85% rejection of data from the original calibration data set. Although the standard errors for each component of the model in estimating the reduced data set are reduced significantly, the large amount of data being removed from the original data set, may lead to a model that does not actually represent the balance behaviour. This means that the accuracy in modelling the range of loads may be greatly reduced, and the calibration matrix may not be a 'good fit' to the data.

Although both 2se and 3se optimisation achieve a lower standard error in all components with less data points being excluded from the original data set (21.5% for 2se and 4.4% for 3se rejection of data from the original data set), such criteria, in excluding certain data points from the original data set, are not recommended because those points being eliminated may represent the actual behaviour of the balance.

With the 329 calibration point data set, the standard error optimisation technique actually increases the standard error of some components of the balance instead of decreasing it. It is believed that this is because, with a much smaller set of calibration data, the effect of removing data points has a more significant effect on the final results compared with a large calibration data set, such as the 1886 data set.

From these observations, it is recommended that standard error optimisation should not be used to increase the accuracy of a particular calibration model because of the elimination of data points, which may represent the actual behaviour of the balance.

7.4.2 Linear Segmentation of Balance Load Range

The accuracy of the calibration model can be increased significantly if it is applied separately within a smaller load range. For example, if the design load range of the drag component of a particular balance is ±1000N, instead of using one calibration

coefficient matrix to describe the entire load range, separate individual calibration matrices could be used for reduced load ranges. This argument is supported by the analysis using the 329 data set. Standard errors obtained from the 329 data set are much lower than the 1886 data set, and this is because the 329 data set only covers loadings from –240N to +240N as compared with the 1886 data set, which covers –1000N to +1000N. This would further imply that the size of each smaller load range should be selected based on the best degree of linearity of the balance loading characteristics within that particular load range.

As shown in Figure 6, for those regions where the balance's loading characteristic is relatively linear, a wider load range can be selected. In the case of non-linear loading characteristics, the size of the load range can be reduced to obtain the same level of accuracy as in the linear region. The main reason to support this linear segmentation method, is because least squares regression methods represent a set of data points using a straight line, and if the data points have a non-linear characteristic the least squares methods are not able to describe the relationship as accurately.

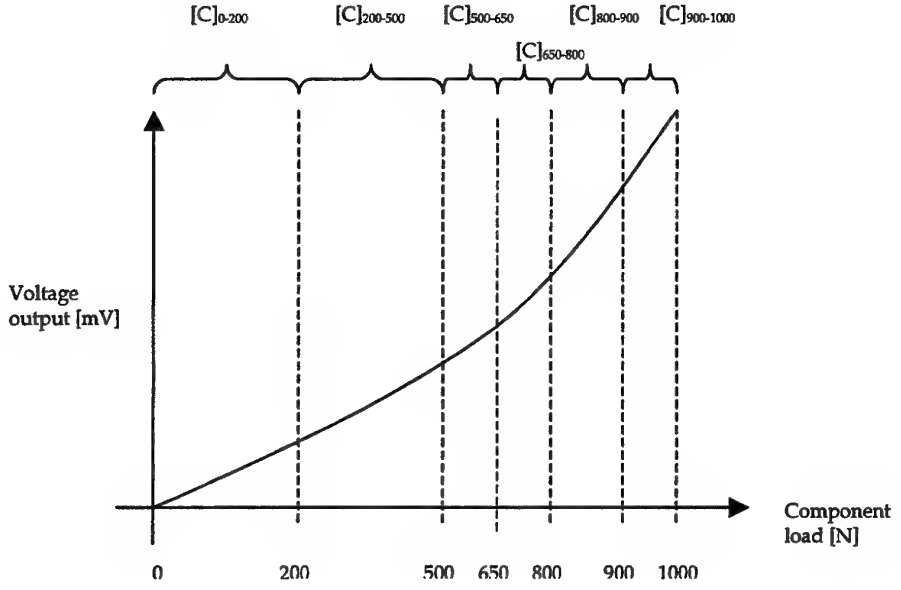


Figure 6. Sub-dividing the design load range of a balance to improve calibration model accuracy

7.4.3 'Zero' Data Filter Optimisation

By eliminating data points which have a value close to zero, no significant effect is observed on the standard errors of each calibration model. In fact, the practice of eliminating data points which are close to zero, may have a negative effect on the

accuracy of the calibration model. This is because a reading close to zero may actually be due to the interaction effect of the balance's components and not background noise.

7.4.4 Chauvenet's Criterion Optimisation

In comparison with the standard error optimisation procedure (see Section 6.2 and Section 7.4.1), Chauvenet's Criterion eliminates an average of 2.2% of data points from each original calibration data set while achieving a significant reduction in the standard errors for each component of the balance. The maximum standard error reduction of 20.5% is achieved in the 1st order 6 coefficient equation as shown in Figure 7. (Refer to Appendix D for the results of the Chauvenet's Criterion for various order calibration equations.)

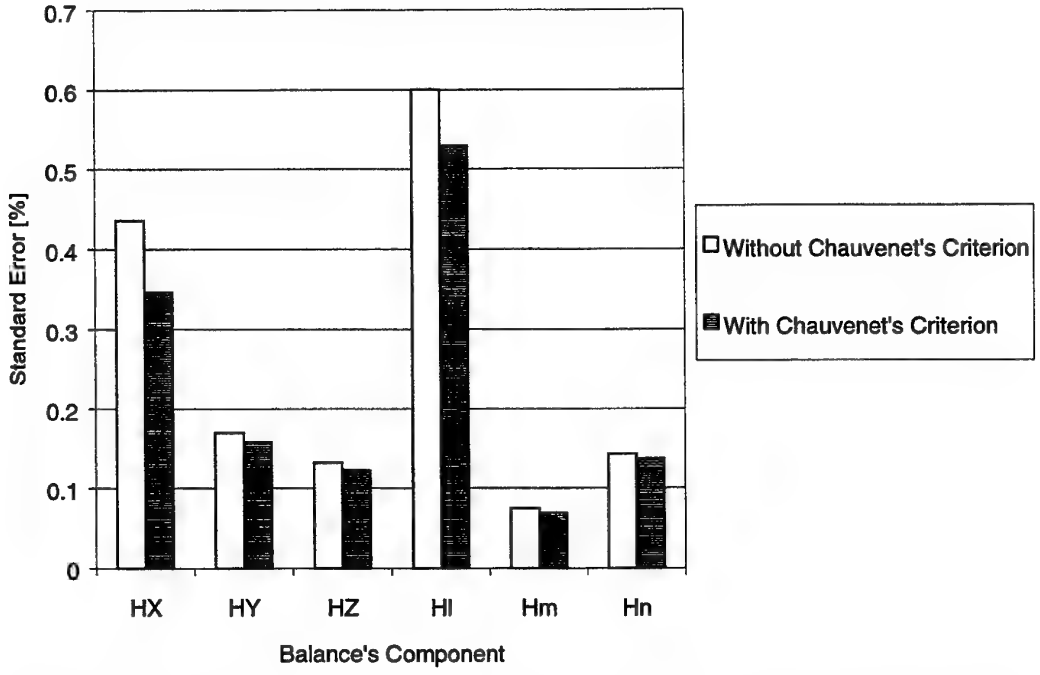


Figure 7. Effect of Chauvenet's Criterion on Standard Errors for [R]=[C][H], 1st order 6 coefficients equation with 1886 data points

Data points eliminated by Chauvenet's Criterion are purely based on statistical analysis with no consideration of the data point's representation of the actual behaviour of the balance. Hence, this optimisation technique should be used with care. It is recommended that data points removed by Chauvenet's Criterion should be documented and reviewed manually to check if some kind of physical and/or theoretical correlation is evident.

7.4.5 Optimised calibration matrix with non-optimised calibration data

Using a standard error optimised calibration matrix with non-optimised calibration data for the reverse calibration process resulted in a lower accuracy (increase in standard errors) compared with the standard error optimised calibration matrix with optimised calibration data. This behaviour is due to the optimised calibration matrix only being able to accurately estimate the loads and moments for the optimised calibration data. If calibration data, other than those within the range of the optimised data, is used, the optimised calibration matrix produces inaccurate load estimations.

Figure 8 and Table 5 show the increase in standard errors when loads and moments are calculated from the optimised (1se optimisation) calibration matrix with non-optimised calibration data for the [H]=[C][R], 2nd order 27 coefficient equation. Similar behaviour is also found in the other calibration models and equations, as shown in Table 6 and Table 7. Standard errors from this optimisation technique increase significantly for all balance components.

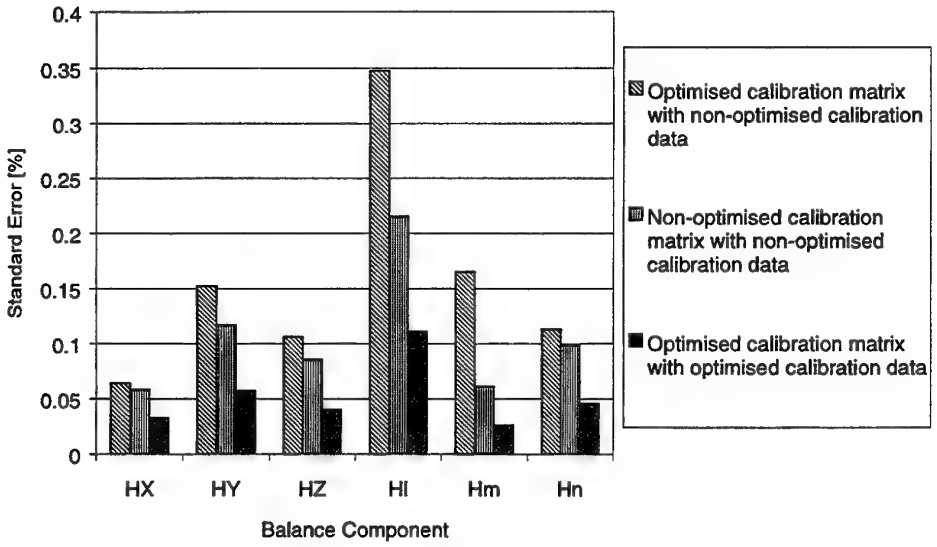


Figure 8. Calibration model: [H]=[C][R], 2nd order equations with 1886 data points

Calibration Matrix	1se Optimised Calibration Data		Non-optimised Calibration Data	
1se optimised calibration matrix	se_1	0.03206%	se_1	0.06598%
	se_2	0.05795%	se_2	0.16158%
	se_3	0.04057%	se_3	0.10531%
	se_4	0.11129%	se_4	0.35291%
	se_5	0.02717%	se_5	0.15965%
	se_6	0.04546%	se_6	0.10913%
Non-optimised calibration matrix			se_1	0.05858%
			se_2	0.11579%
			se_3	0.08537%
			se_4	0.21562%
			se_5	0.06154%
			se_6	0.09877%

Table 5. Standard errors for 1se optimised calibration matrix with non-optimised calibration data for the [R]=[C][H], 2nd order calibration equation with 1886 data points

Calibration Matrix	1se Optimised Calibration Data		Non-optimised Calibration Data	
1se optimised calibration matrix	se_1	0.03264%	se_1	0.06421%
	se_2	0.05706%	se_2	0.15216%
	se_3	0.04034%	se_3	0.10593%
	se_4	0.11079%	se_4	0.34769%
	se_5	0.02618%	se_5	0.16481%
	se_6	0.04572%	se_6	0.11289%
Non-optimised calibration matrix			se_1	0.05860%
			se_2	0.11663%
			se_3	0.08537%
			se_4	0.21537%
			se_5	0.06149%
			se_6	0.09848%

Table 6. Standard errors for 1se optimised calibration matrix with non-optimised calibration data for the [H]=[C][R], 2nd order calibration equation with 1886 data points

Calibration Matrix	1se Optimised Calibration Data		Non-optimised Calibration Data	
1se optimised calibration matrix	se_1	0.03247%	se_1	0.06639%
	se_2	0.05742%	se_2	0.15661%
	se_3	0.04042%	se_3	0.10463%
	se_4	0.11101%	se_4	0.34818%
	se_5	0.02667%	se_5	0.16417%
	se_6	0.04556%	se_6	0.11018%
Non-optimised calibration matrix			se_1	0.05858%
			se_2	0.11579%
			se_3	0.08537%
			se_4	0.21562%
			se_5	0.06154%
			se_6	0.09877%

Table 7. Standard errors for 1se optimised calibration matrix with non-optimised calibration data for the [H]=[C][R-H], 2nd order calibration equation with 1886 data points

For the Chauvenet's Criterion optimisation technique, no significant improvement in accuracy was found for the optimised calibration matrix with non-optimised calibration data (see Figure 9), compared with the non-optimised calibration results. This is because the Chauvenet's Criterion optimised calibration matrix estimates the loads only within the optimised calibration data set, and because very few points are eliminated, the optimised load data set is similar to the full data set. This behaviour can also be observed for other calibration models, as shown in Table 8.

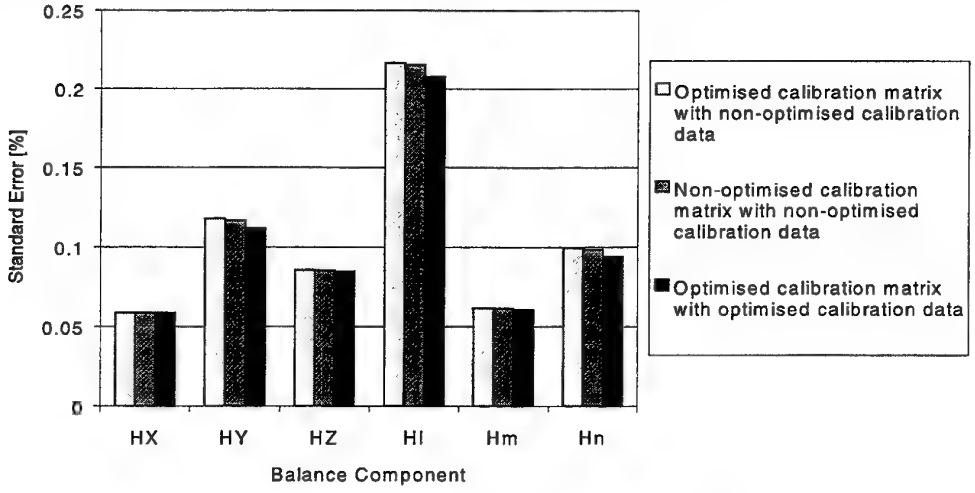


Figure 9. Chauvenet's Criterion optimisation for calibration model: [H]=[C][R], 2nd order equations with 1886 data points

Calibration Matrix	Chauvenet's Criterion Optimised Calibration Data		Non-optimised Calibration Data	
[R]=[C][H], 2nd order, 27 coefficient equation				
Chauvenet's Criterion optimised calibration matrix	se_1	0.05862%	se_1	0.05858%
	se_2	0.11711%	se_2	0.11579%
	se_3	0.08546%	se_3	0.08537%
	se_4	0.21672%	se_4	0.21562%
	se_5	0.06168%	se_5	0.06154%
	se_6	0.09957%	se_6	0.09877%
[H]=[C][R], 2nd order, 27 coefficient equation				
Chauvenet's Criterion optimised calibration matrix	se_1	0.05866%	se_1	0.05860%
	se_2	0.11798%	se_2	0.11663%
	se_3	0.08548%	se_3	0.08537%
	se_4	0.21654%	se_4	0.21537%
	se_5	0.06161%	se_5	0.06149%
	se_6	0.09918%	se_6	0.09848%
[H]=[C][R-H], 2nd order, 27 coefficient equation				
Chauvenet's Criterion optimised calibration matrix	se_1	0.05862%	se_1	0.05858%
	se_2	0.11713%	se_2	0.11579%
	se_3	0.08546%	se_3	0.08537%
	se_4	0.21674%	se_4	0.21562%
	se_5	0.06168%	se_5	0.06154%
	se_6	0.09956%	se_6	0.09877%

Table 8. Standard errors for Chauvenet's Criterion optimised calibration matrix with optimised and non-optimised calibration data for the 2nd order calibration equation with 1886 data points

In summary, it is recommended that the optimisation techniques described in this report should be used with utmost care, and if any, Chauvenet's Criterion, provides the best results without degradation in the ability to represent the actual balance behaviour.

8 *CALIB* – The Computer Program

A computer program, written in the C and X/*Motif* programming languages has been developed for use in the wind tunnels at AMRL. The aim of the computer program is to allow effective and efficient analysis of various balance calibration mathematical models, and to enable real-time conversion from balance voltage output to the corresponding load experienced by the balance during wind tunnel tests.

8.1 Program structure

A modular programming structure was used to ensure a high level of flexibility in the code. The advantages of such a structure ensures minimum modification of the existing data acquisition code if a new balance calibration model is integrated into the current data acquisition system. For example, a third order regression method for balance coefficient calculations can be added easily to the existing code as an individual function, and this would not require any modification to the existing code. A flow chart for the program *CALIB* is given in Section 8.3.

All variables used in the program are stored in a data structure, hence the code can be easily integrated with the current data acquisition system.

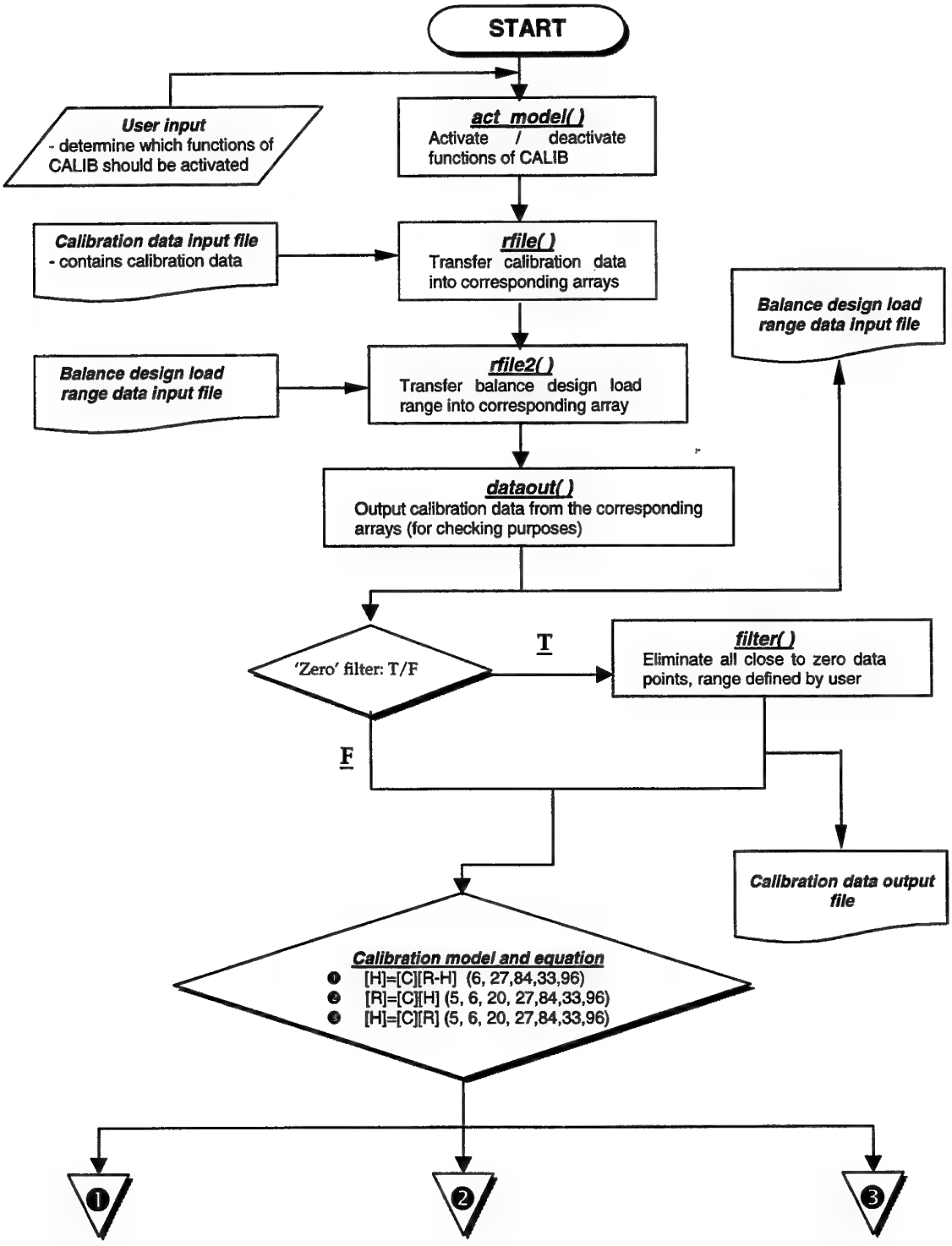
8.2 Program operation

The program requires a balance calibration data input file which must be in the format of $\{H_1.....H_6, R_1.....R_6\}$ and a balance design load range input file, listing the maximum design load range for a particular balance under investigation (see Appendix B.1 and B.2 for samples of these data input files).

A data output file shown in Appendix B.3 is generated by the program. It contains details of the calibration model, optimisation details, the balance calibration coefficients, standard errors and the coefficient of multiple correlation.

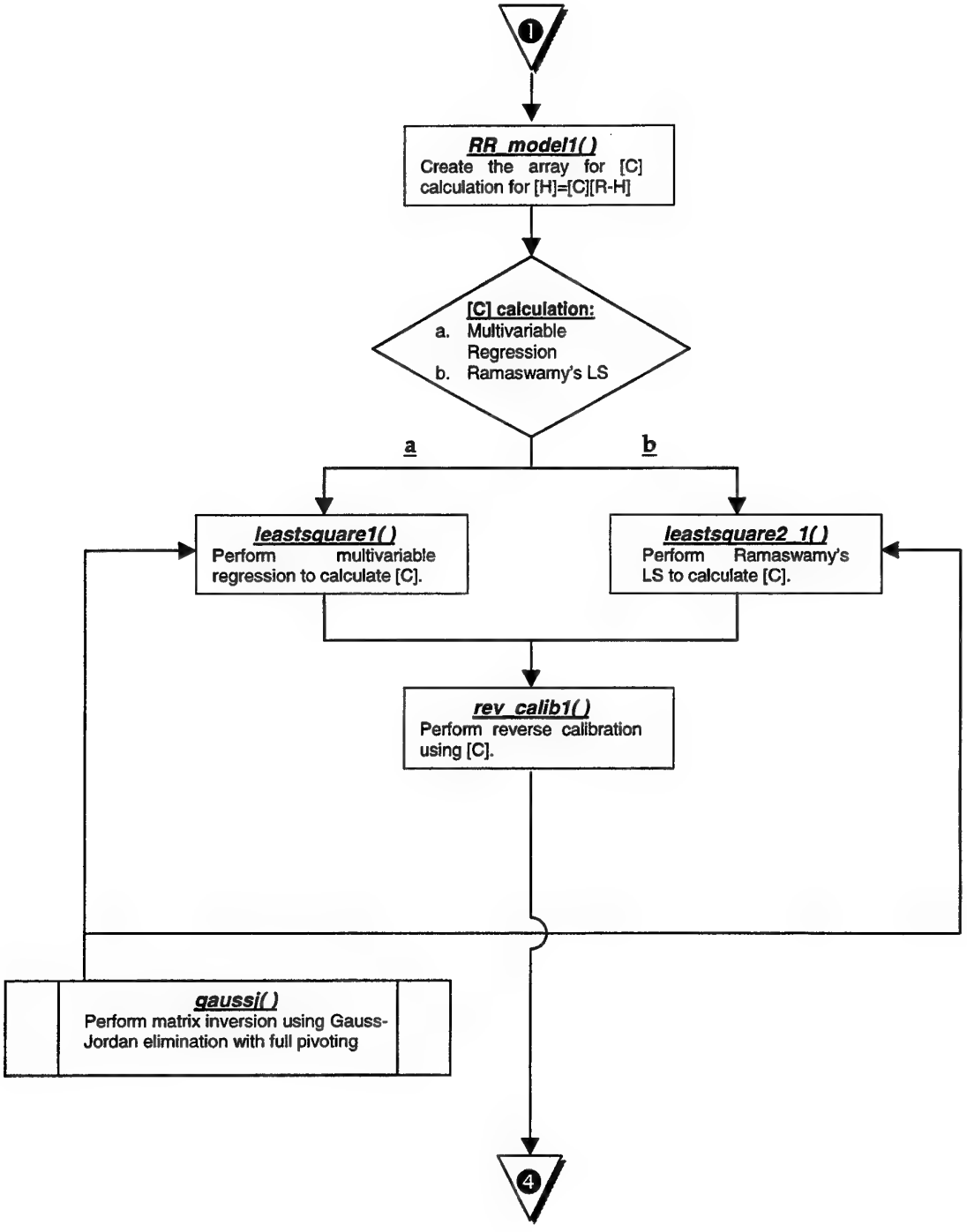
The graphical user interface (GUI) (see Appendix C) provides a user friendly environment for the end users. The GUI clearly lists all the available options in a single window, and users may select the calibration equation and appropriate options for the analysis.

8.3 *CALIB*'s Flow Chart



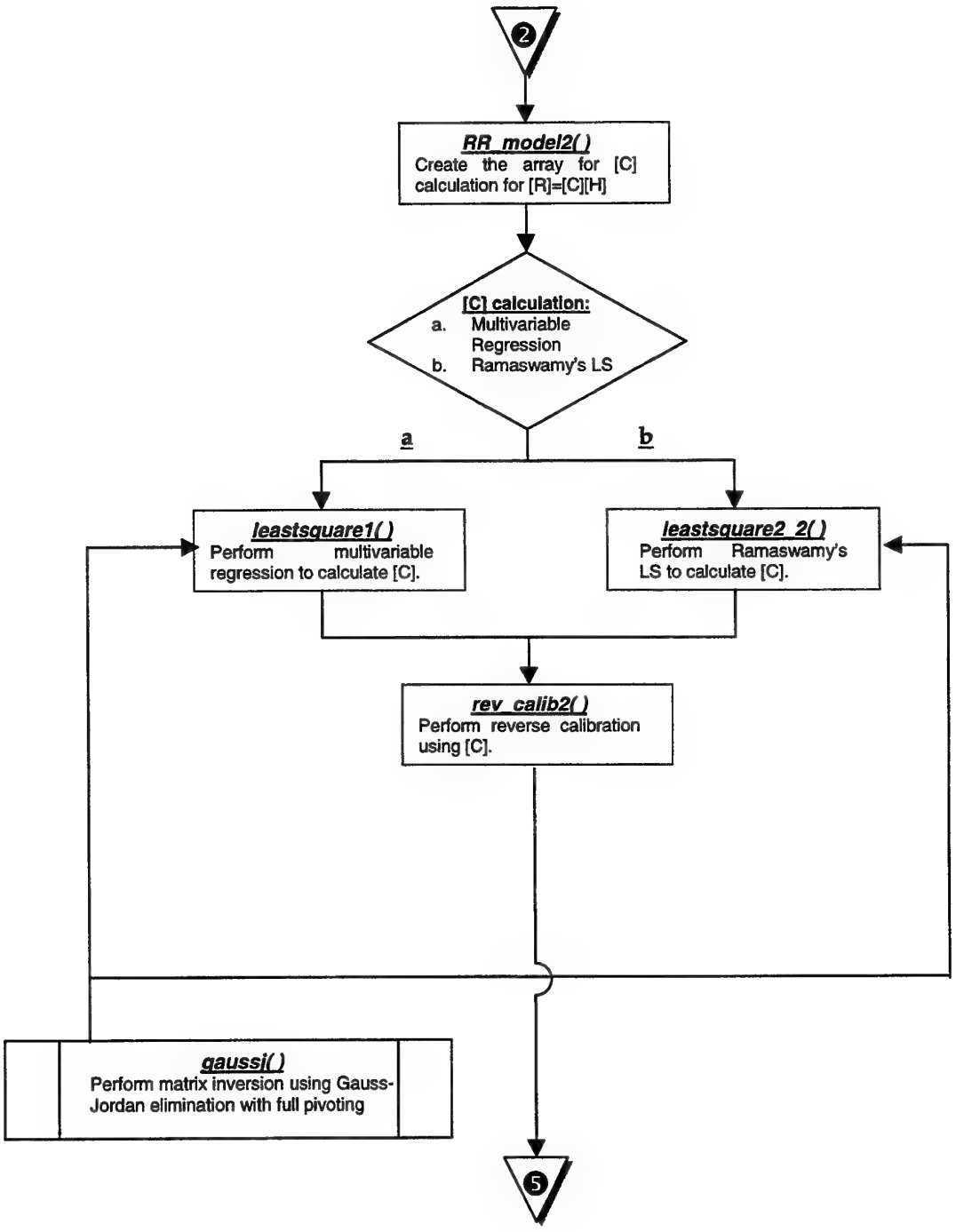
(CALIB's flow chart contd)

This part of the flow chart represents the calculation of the calibration coefficients and reverse calibration for model: [H] = [C][R-H] (6, 27, 84, 33, 96 coefficients)



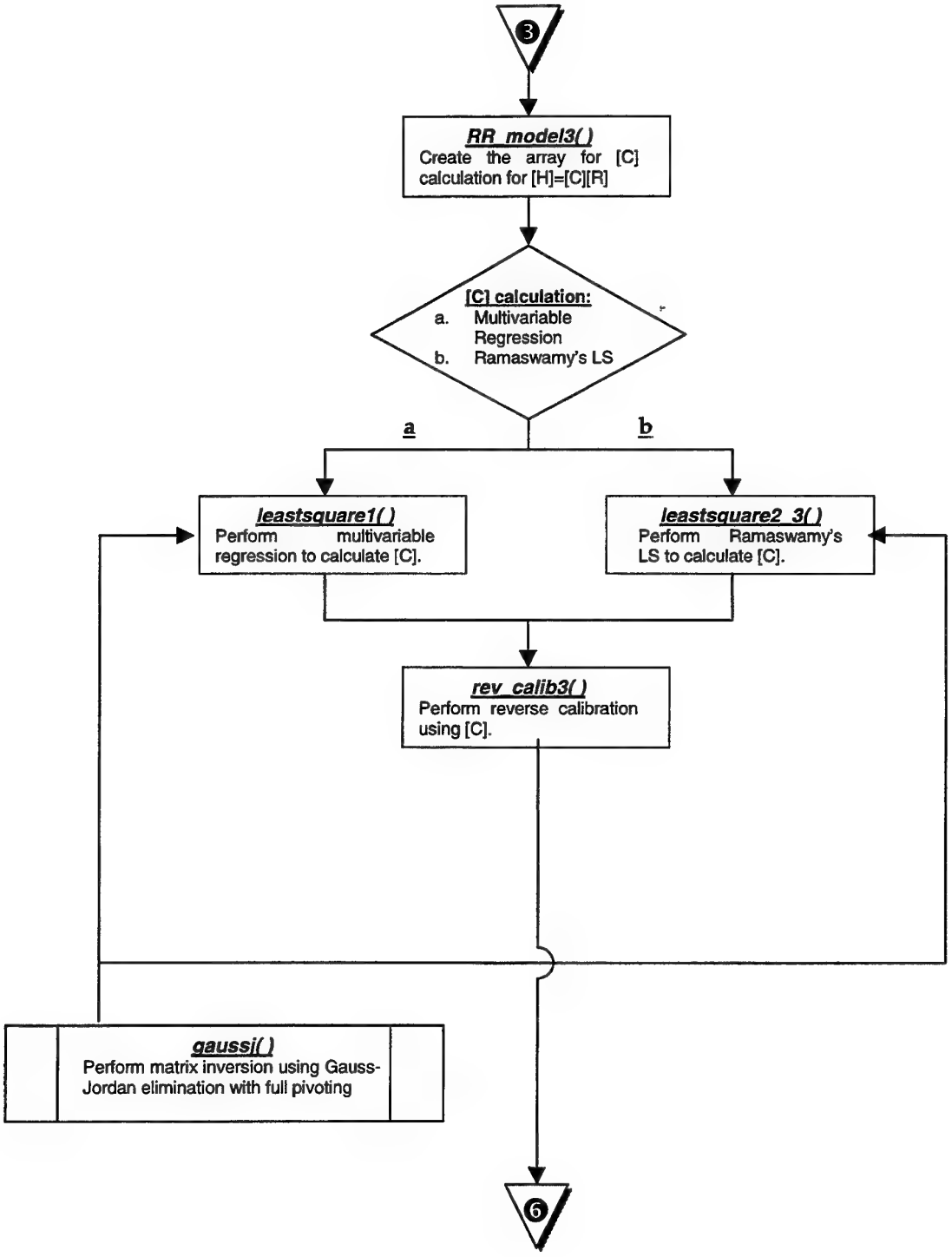
(CALIB's flow chart contd)

This part of the flow chart represents the calculation of the calibration coefficients and reverse calibration for model: [R] = [C][H] (5, 6, 20, 27, 84, 33, 96 coefficients)

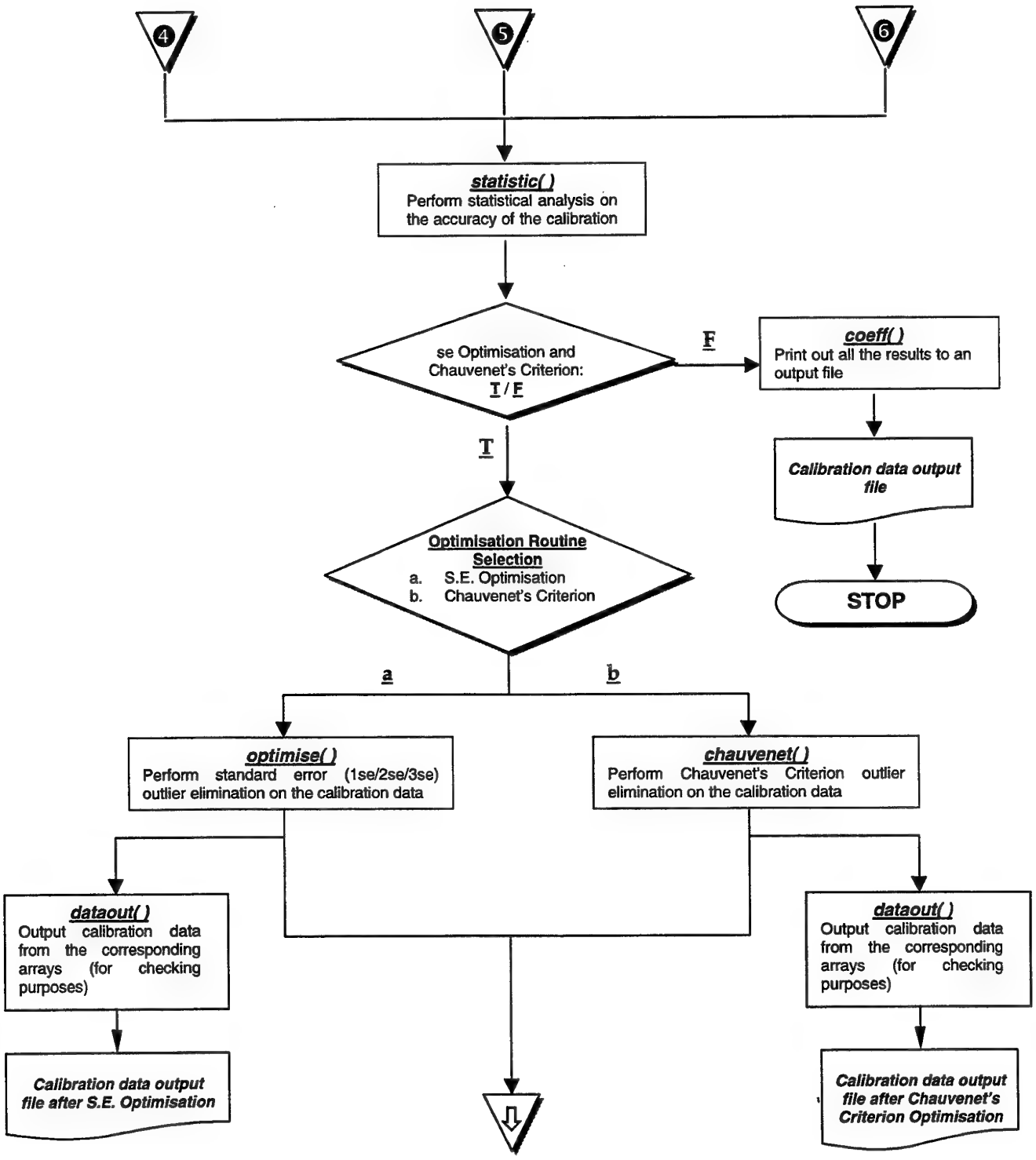


(CALIB's flow chart contd)

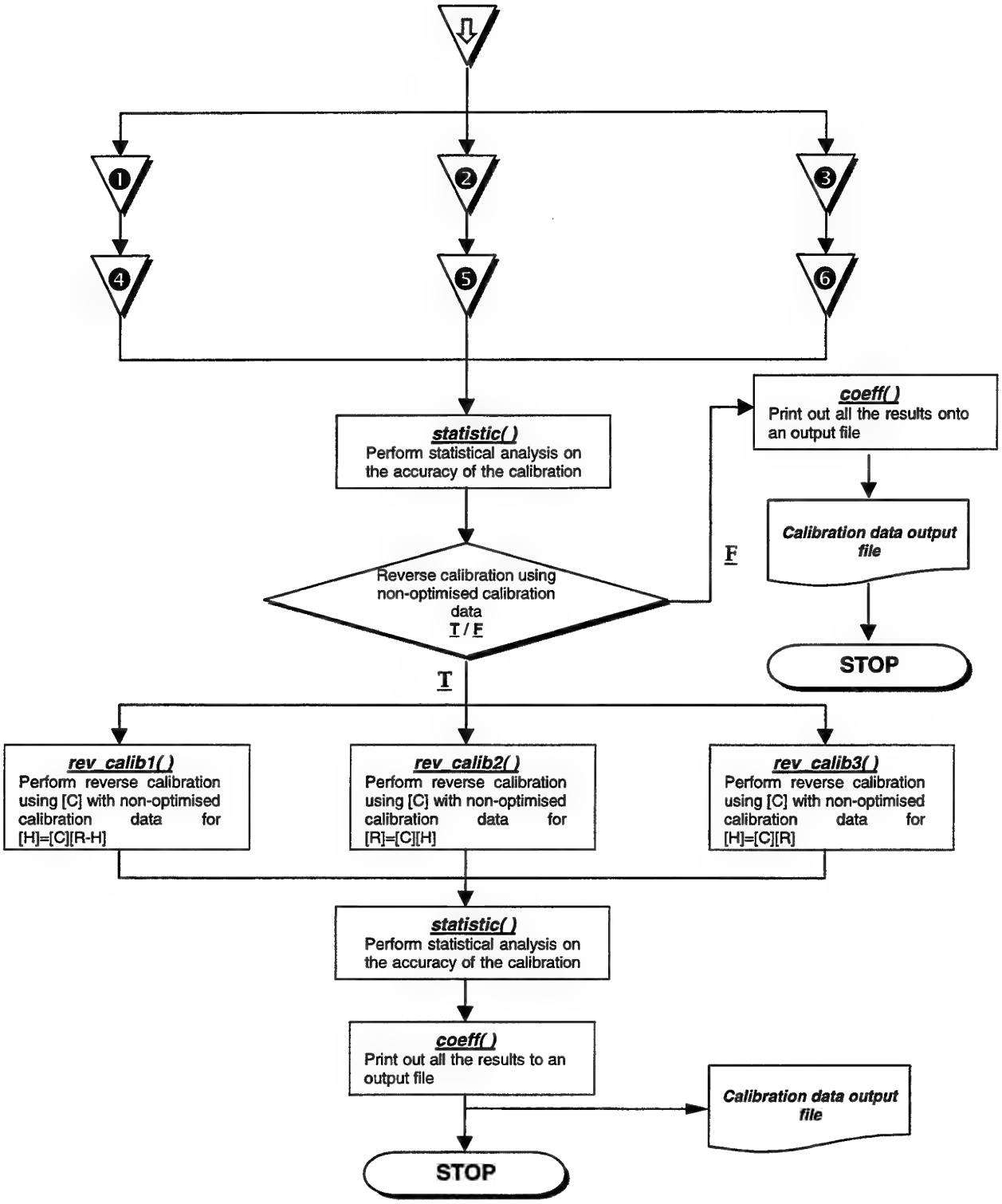
This part of the flow chart represents the calculation of the calibration coefficients and reverse calibration for model: [H] = [C][R] (5, 6, 20, 27, 84, 33, 96 coefficients)



(CALIB's flow chart contd)



(CALIB's flow chart contd)



9 Conclusion

All three calibration models have very similar behaviour in terms of accuracy. As the order of the calibration equation increases, so too does the accuracy of the estimated forces and moments. A small, but insignificant improvement in accuracy was found between the second order (27 coefficients) and third order (33 coefficients) calibration equations. The small difference between these second and third order results may primarily be due to background interference rather than actual balance component interaction. A significant improvement in accuracy was found by using the 2nd order 84 and 3rd order 96 coefficient balance calibration equations.

Although a significant improvement in accuracy can be achieved by various data optimisation techniques, it is recommended that data optimisation techniques which require the elimination of calibration data should not be used. This is because those data points being eliminated may actually represent the true behaviour of the balance. The results obtained from the optimised calibration matrix with the non-optimised calibration data set show a reduction in accuracy of the estimated loads. This is due to the inability of the optimised calibration matrix to estimate the loads for those calibration data points which were eliminated during the optimisation process. Of the optimisation techniques presented, Chauvenet's Criterion is the most suitable as it provides a reduction in standard error by eliminating a minimum number of data points.

To avoid the elimination of calibration data points whilst aiming to achieve a high level of accuracy for the calibration model, it is suggested that the calibration matrix should only cover a sub-divided load range within the balance design load range. This would lead to the use of more than one balance calibration matrix to cover the required load range of the balance.

Due to the nature of the least squares regression methods, an adequate number of calibration data points should be provided for the calculation of the calibration coefficients. It was found that the reverse calibration method might fail to converge if an inadequate number of calibration data points were provided.

The computer program, *CALIB*, allows an efficient and effective way to apply various calibration equations described in this report with different combinations of data optimisation techniques. This flexibility allows the user to select the most appropriate order of the calibration equation and data optimisation technique based on the test requirements.

10 Acknowledgements

The authors would like to thank the following individuals and indicate their area of contribution.

Dr Stephen Lam - software programming in the *C* and X/*Motif* programming languages;
- optimisation technique for strain gauge balance calibration.

Dr Neil Matheson - providing comments and advice for this investigation.

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Appendix A Balance Calibration Models

A complete list of all the balance calibration mathematical models are listed below, where $i = 1 \ldots 6$ (unless otherwise specified).

A.1 Calibration Model: [R] = [C][H]

A.1.1 First order, 6 component equation with 6 coefficients

$$R_i = C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6$$

A.1.2 First order, 5 component equation with 5 coefficients

$$R_i = C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6$$

where, $i = 2 \ldots 5$

A.1.3 Second order, 6 component equation with 27 coefficients

$$\begin{aligned} R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6 \\ & + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2} \\ & + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6 \\ & + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6 \\ & + C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6 \\ & + C_{i,45}H_4H_5 + C_{i,46}H_4H_6 \\ & + C_{i,56}H_5H_6 \end{aligned}$$

A.1.4 Second order, 5 component equation with 20 coefficients

$$\begin{aligned} R_i = {} & C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6 \\ & + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2} \\ & + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6 \\ & + C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6 \\ & + C_{i,45}H_4H_5 + C_{i,46}H_4H_6 \\ & + C_{i,56}H_5H_6 \end{aligned}$$

where, $i = 2 \ldots 5$

A.1.5 Third order, 6 component equation with 33 coefficients

$$
\begin{aligned}
R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6 \\
& + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2} \\
& + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6 \\
& + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6 \\
& + C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6 \\
& + C_{i,45}H_4H_5 + C_{i,46}H_4H_6 \\
& + C_{i,56}H_5H_6 \\
& + C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + C_{i,333}H_3^{\,3} + C_{i,444}H_4^{\,3} + C_{i,555}H_5^{\,3}C_{i,666}H_6^{\,3}
\end{aligned}
$$

A.1.6 Second order, 6 component equation with 84 coefficients

$$
\begin{aligned}
R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6 \\
& + C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + C_{i,|3|}|H_3| + C_{i,|4|}|H_4| + C_{i,|5|}|H_5| + C_{i,|6|}|H_6| \\
& + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2} \\
& + C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + C_{i,3|3|}H_3|H_3| + C_{i,4|4|}H_4|H_4| + C_{i,5|5|}H_5|H_5| + C_{i,6|6|}H_6|H_6| \\
& + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6 \\
& + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6 \\
& + C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6 \\
& + C_{i,45}H_4H_5 + C_{i,46}H_4H_6 \\
& + C_{i,56}H_5H_6 \\
& + C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + C_{i,|14|}|H_1H_4| + C_{i,|15|}|H_1H_5| + C_{i,|16|}|H_1H_6| \\
& + C_{i,|23|}|H_2H_3| + C_{i,|24|}|H_2H_4| + C_{i,|25|}|H_2H_5| + C_{i,|26|}|H_2H_6| \\
& + C_{i,|34|}|H_3H_4| + C_{i,|35|}|H_3H_5| + C_{i,|36|}|H_3H_6| \\
& + C_{i,|45|}|H_4H_5| + C_{i,|46|}|H_4H_6| \\
& + C_{i,|56|}|H_5H_6| \\
& + C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + C_{i,1|4|}H_1|H_4| + C_{i,1|5|}H_1|H_5| + C_{i,1|6|}H_1|H_6| \\
& + C_{i,2|3|}H_2|H_3| + C_{i,2|4|}H_2|H_4| + C_{i,2|5|}H_2|H_5| + C_{i,2|6|}H_2|H_6| \\
& + C_{i,3|4|}H_3|H_4| + C_{i,3|5|}H_3|H_5| + C_{i,3|6|}H_3|H_6| \\
& + C_{i,4|5|}H_4|H_5| + C_{i,4|6|}H_4|H_6| \\
& + C_{i,5|6|}H_5|H_6| \\
& + C_{i,|1|2|}|H_1|H_2 + C_{i,|1|3|}|H_1|H_3 + C_{i,|1|4|}|H_1|H_4 + C_{i,|1|5|}|H_1|H_5 + C_{i,|1|6|}|H_1|H_6 \\
& + C_{i,|2|3|}|H_2|H_3 + C_{i,|2|4|}|H_2|H_4 + C_{i,|2|5|}|H_2|H_5 + C_{i,|2|6|}|H_2|H_6 \\
& + C_{i,|3|4|}|H_3|H_4 + C_{i,|3|5|}|H_3|H_5 + C_{i,|3|6|}|H_3|H_6 \\
& + C_{i,|4|5|}|H_4|H_5 + C_{i,|4|6|}|H_4|H_6 \\
& + C_{i,|5|6|}|H_5|H_6
\end{aligned}
$$

A.1.7 Third order, 6 component equation with 96 coefficients

$$
\begin{aligned}
R_i = {} & C_{i,1}H_1 + C_{i,2}H_2 + C_{i,3}H_3 + C_{i,4}H_4 + C_{i,5}H_5 + C_{i,6}H_6 \\
& + C_{i,|1|}|H_1| + C_{i,|2|}|H_2| + C_{i,|3|}|H_3| + C_{i,|4|}|H_4| + C_{i,|5|}|H_5| + C_{i,|6|}|H_6| \\
& + C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2} \\
& + C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + C_{i,3|3|}H_3|H_3| + C_{i,4|4|}H_4|H_4| + C_{i,5|5|}H_5|H_5| + C_{i,6|6|}H_6|H_6| \\
& + C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6 \\
& + C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6 \\
& + C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6 \\
& + C_{i,45}H_4H_5 + C_{i,46}H_4H_6 \\
& + C_{i,56}H_5H_6 \\
& + C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + C_{i,|14|}|H_1H_4| + C_{i,|15|}|H_1H_5| + C_{i,|16|}|H_1H_6| \\
& + C_{i,|23|}|H_2H_3| + C_{i,|24|}|H_2H_4| + C_{i,|25|}|H_2H_5| + C_{i,|26|}|H_2H_6| \\
& + C_{i,|34|}|H_3H_4| + C_{i,|35|}|H_3H_5| + C_{i,|36|}|H_3H_6| \\
& + C_{i,|45|}|H_4H_5| + C_{i,|46|}|H_4H_6| \\
& + C_{i,|56|}|H_5H_6| \\
& + C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + C_{i,1|4|}H_1|H_4| + C_{i,1|5|}H_1|H_5| + C_{i,1|6|}H_1|H_6| \\
& + C_{i,2|3|}H_2|H_3| + C_{i,2|4|}H_2|H_4| + C_{i,2|5|}H_2|H_5| + C_{i,2|6|}H_2|H_6| \\
& + C_{i,3|4|}H_3|H_4| + C_{i,3|5|}H_3|H_5| + C_{i,3|6|}H_3|H_6| \\
& + C_{i,4|5|}H_4|H_5| + C_{i,4|6|}H_4|H_6| \\
& + C_{i,5|6|}H_5|H_6| \\
& + C_{i,|1|2|}|H_1|H_2 + C_{i,|1|3|}|H_1|H_3 + C_{i,|1|4|}|H_1|H_4 + C_{i,|1|5|}|H_1|H_5 + C_{i,|2|6|}|H_1|H_6 \\
& + C_{i,|2|3|}|H_2|H_3 + C_{i,|2|4|}|H_2|H_4 + C_{i,|2|5|}|H_2|H_5 + C_{i,|2|6|}|H_2|H_6 \\
& + C_{i,|3|4|}|H_3|H_4 + C_{i,|3|5|}|H_3|H_5 + C_{i,|3|6|}|H_3|H_6 \\
& + C_{i,|4|5|}|H_4|H_5 + C_{i,|4|6|}|H_4|H_6 \\
& + C_{i,|5|6|}|H_5|H_6 \\
& + C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + C_{i,333}H_3^{\,3} + C_{i,444}H_4^{\,3} + C_{i,555}H_5^{\,3}C_{i,666} + H_6^{\,3} \\
& + C_{i,|111|}|H_1^{\,3}| + C_{i,|222|}|H_2^{\,3}| + C_{i,|333|}|H_3^{\,3}| + C_{i,|444|}|H_4^{\,3}| + C_{i,|555|}|H_5^{\,3}| + C_{i,|666|}|H_6^{\,3}|
\end{aligned}
$$

A.2 Calibration Model: [H] = [C][R]

A.2.1 First order, 6 component equation with 6 coefficients

$$H_i = C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6$$

A.2.2 First order, 5 component equation with 5 coefficients

$$H_i = C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6$$

where, $i = 2...5$

A.2.3 Second order, 6 component equation with 27 coefficients

$$\begin{aligned} H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6 \\ & + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + C_{i,44}R_4^{\,2} + C_{i,55}R_5^{\,2} + C_{i,66}R_6^{\,2} \\ & + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + C_{i,14}R_1R_4 + C_{i,15}R_1R_5 + C_{i,16}R_1R_6 \\ & + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + C_{i,25}R_2R_5 + C_{i,26}R_2R_6 \\ & + C_{i,34}R_3R_4 + C_{i,35}R_3R_5 + C_{i,36}R_3R_6 \\ & + C_{i,45}R_4R_5 + C_{i,46}R_4R_6 \\ & + C_{i,56}R_5R_6 \end{aligned}$$

A.2.4 Second order, 5 component equation with 20 coefficients

$$\begin{aligned} H_i = {} & C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6 \\ & + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + C_{i,44}R_4^{\,2} + C_{i,55}R_5^{\,2} + C_{i,66}R_6^{\,2} \\ & + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + C_{i,25}R_2R_5 + C_{i,26}R_2R_6 \\ & + C_{i,34}R_3R_4 + C_{i,35}R_3R_5 + C_{i,36}R_3R_6 \\ & + C_{i,45}R_4R_5 + C_{i,46}R_4R_6 \\ & + C_{i,56}R_5R_6 \end{aligned}$$

where, $i = 2...5$

A.2.5 Third order, 6 component equation with 33 coefficients

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6 \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + C_{i,44}R_4^{\,2} + C_{i,55}R_5^{\,2} + C_{i,66}R_6^{\,2} \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + C_{i,14}R_1R_4 + C_{i,15}R_1R_5 + C_{i,16}R_1R_6 \\
& + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + C_{i,25}R_2R_5 + C_{i,26}R_2R_6 \\
& + C_{i,34}R_3R_4 + C_{i,35}R_3R_5 + C_{i,36}R_3R_6 \\
& + C_{i,45}R_4R_5 + C_{i,46}R_4R_6 \\
& + C_{i,56}R_5R_6 \\
& + C_{i,111}R_1^{\,3} + C_{i,222}R_2^{\,3} + C_{i,333}R_3^{\,3} + C_{i,444}R_4^{\,3} + C_{i,555}R_5^{\,3}C_{i,666}R_6^{\,3}
\end{aligned}
$$

A.2.6 Second order, 6 component equation with 84 coefficients

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6 \\
& + C_{i,|1|}|R_1| + C_{i,|2|}|R_2| + C_{i,|3|}|R_3| + C_{i,|4|}|R_4| + C_{i,|5|}|R_5| + C_{i,|6|}|R_6| \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + C_{i,44}R_4^{\,2} + C_{i,55}R_5^{\,2} + C_{i,66}R_6^{\,2} \\
& + C_{i,1|1|}R_1|R_1| + C_{i,2|2|}R_2|R_2| + C_{i,3|3|}R_3|R_3| + C_{i,4|4|}R_4|R_4| + C_{i,5|5|}R_5|R_5| + C_{i,6|6|}R_6|R_6| \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + C_{i,14}R_1R_4 + C_{i,15}R_1R_5 + C_{i,16}R_1R_6 \\
& + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + C_{i,25}R_2R_5 + C_{i,26}R_2R_6 \\
& + C_{i,34}R_3R_4 + C_{i,35}R_3R_5 + C_{i,36}R_3R_6 \\
& + C_{i,45}R_4R_5 + C_{i,46}R_4R_6 \\
& + C_{i,56}R_5R_6 \\
& + C_{i,|12|}|R_1R_2| + C_{i,|13|}|R_1R_3| + C_{i,|14|}|R_1R_4| + C_{i,|15|}|R_1R_5| + C_{i,|16|}|R_1R_6| \\
& + C_{i,|23|}|R_2R_3| + C_{i,|24|}|R_2R_4| + C_{i,|25|}|R_2R_5| + C_{i,|26|}|R_2R_6| \\
& + C_{i,|34|}|R_3R_4| + C_{i,|35|}|R_3R_5| + C_{i,|36|}|R_3R_6| \\
& + C_{i,|45|}|R_4R_5| + C_{i,|46|}|R_4R_6| \\
& + C_{i,|56|}|R_5R_6| \\
& + C_{i,1|2|}R_1|R_2| + C_{i,1|3|}R_1|R_3| + C_{i,1|4|}R_1|R_4| + C_{i,1|5|}R_1|R_5| + C_{i,1|6|}R_1|R_6| \\
& + C_{i,2|3|}R_2|R_3| + C_{i,2|4|}R_2|R_4| + C_{i,2|5|}R_2|R_5| + C_{i,2|6|}R_2|R_6| \\
& + C_{i,3|4|}R_3|R_4| + C_{i,3|5|}R_3|R_5| + C_{i,3|6|}R_3|R_6| \\
& + C_{i,4|5|}R_4|R_5| + C_{i,4|6|}R_4|R_6| \\
& + C_{i,5|6|}R_5|R_6| \\
& + C_{i,|1|2|}|R_1|R_2 + C_{i,|1|3|}|R_1|R_3 + C_{i,|1|4|}|R_1|R_4 + C_{i,|1|5|}|R_1|R_5 + C_{i,|1|6|}|R_1|R_6 \\
& + C_{i,|2|3|}|R_2|R_3 + C_{i,|2|4|}|R_2|R_4 + C_{i,|2|5|}|R_2|R_5 + C_{i,|2|6|}|R_2|R_6 \\
& + C_{i,|3|4|}|R_3|R_4 + C_{i,|3|5|}|R_3|R_5 + C_{i,|3|6|}|R_3|R_6 \\
& + C_{i,|4|5|}|R_4|R_5 + C_{i,|4|6|}|R_4|R_6 \\
& + C_{i,|5|6|}|R_5|R_6
\end{aligned}
$$

A.2.7 Third order, 6 component equation with 96 coefficients

$$
\begin{aligned}
H_i = {} & C_{i,1}R_1 + C_{i,2}R_2 + C_{i,3}R_3 + C_{i,4}R_4 + C_{i,5}R_5 + C_{i,6}R_6 \\
& + C_{i,|1|}|R_1| + C_{i,|2|}|R_2| + C_{i,|3|}|R_3| + C_{i,|4|}|R_4| + C_{i,|5|}|R_5| + C_{i,|6|}|R_6| \\
& + C_{i,11}R_1^{\,2} + C_{i,22}R_2^{\,2} + C_{i,33}R_3^{\,2} + C_{i,44}R_4^{\,2} + C_{i,55}R_5^{\,2} + C_{i,66}R_6^{\,2} \\
& + C_{i,1|1|}R_1|R_1| + C_{i,2|2|}R_2|R_2| + C_{i,3|3|}R_3|R_3| + C_{i,4|4|}R_4|R_4| + C_{i,5|5|}R_5|R_5| + C_{i,6|6|}R_6|R_6| \\
& + C_{i,12}R_1R_2 + C_{i,13}R_1R_3 + C_{i,14}R_1R_4 + C_{i,15}R_1R_5 + C_{i,16}R_1R_6 \\
& + C_{i,23}R_2R_3 + C_{i,24}R_2R_4 + C_{i,25}R_2R_5 + C_{i,26}R_2R_6 \\
& + C_{i,34}R_3R_4 + C_{i,35}R_3R_5 + C_{i,36}R_3R_6 \\
& + C_{i,45}R_4R_5 + C_{i,46}R_4R_6 \\
& + C_{i,56}R_5R_6 \\
& + C_{i,|12|}|R_1R_2| + C_{i,|13|}|R_1R_3| + C_{i,|14|}|R_1R_4| + C_{i,|15|}|R_1R_5| + C_{i,|16|}|R_1R_6| \\
& + C_{i,|23|}|R_2R_3| + C_{i,|24|}|R_2R_4| + C_{i,|25|}|R_2R_5| + C_{i,|26|}|R_2R_6| \\
& + C_{i,|34|}|R_3R_4| + C_{i,|35|}|R_3R_5| + C_{i,|36|}|R_3R_6| \\
& + C_{i,|45|}|R_4R_5| + C_{i,|46|}|R_4R_6| \\
& + C_{i,|56|}|R_5R_6| \\
& + C_{i,1|2|}R_1|R_2| + C_{i,1|3|}R_1|R_3| + C_{i,1|4|}R_1|R_4| + C_{i,1|5|}R_1|R_5| + C_{i,1|6|}R_1|R_6| \\
& + C_{i,2|3|}R_2|R_3| + C_{i,2|4|}R_2|R_4| + C_{i,2|5|}R_2|R_5| + C_{i,2|6|}R_2|R_6| \\
& + C_{i,3|4|}R_3|R_4| + C_{i,3|5|}R_3|R_5| + C_{i,3|6|}R_3|R_6| \\
& + C_{i,4|5|}R_4|R_5| + C_{i,4|6|}R_4|R_6| \\
& + C_{i,5|6|}R_5|R_6| \\
& + C_{i,|1|2|}R_1|R_2 + C_{i,|1|3|}R_1|R_3 + C_{i,|1|4|}R_1|R_4 + C_{i,|1|5|}R_1|R_5 + C_{i,|1|6|}R_1|R_6 \\
& + C_{i,|2|3|}R_2|R_3 + C_{i,|2|4|}R_2|R_4 + C_{i,|2|5|}R_2|R_5 + C_{i,|2|6|}R_2|R_6 \\
& + C_{i,|3|4|}R_3|R_4 + C_{i,|3|5|}R_3|R_5 + C_{i,|3|6|}R_3|R_6 \\
& + C_{i,|4|5|}R_4|R_5 + C_{i,|4|6|}R_4|R_6 \\
& + C_{i,|5|6|}R_5|R_6 \\
& + C_{i,111}R_1^{\,3} + C_{i,222}R_2^{\,3} + C_{i,333}R_3^{\,3} + C_{i,444}R_4^{\,3} + C_{i,555}R_5^{\,3} + C_{i,666}R_6^{\,3} \\
& + C_{i,|111|}|R_1^{\,3}| + C_{i,|222|}|R_2^{\,3}| + C_{i,|333|}|R_3^{\,3}| + C_{i,|444|}|R_4^{\,3}| + C_{i,|555|}|R_5^{\,3}| + C_{i,|666|}|R_6^{\,3}|
\end{aligned}
$$

A.3 Calibration Model: [H] = [C][R-H]

A.3.1 First order, 6 component equation with 6 coefficients

$$H_i = C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6$$

A.3.2 Second order, 6 component equation with 27 coefficients

$$\begin{aligned} H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6 \\ & - \left(C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2}\right) \\ & - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6\right) \\ & - \left(C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6\right) \\ & - \left(C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6\right) \\ & - \left(C_{i,45}H_4H_5 + C_{i,46}H_4H_6\right) \\ & - \left(C_{i,56}H_5H_6\right) \end{aligned}$$

A.3.3 Third order, 6 component equation with 33 coefficients

$$\begin{aligned} H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6 \\ & - \left(C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2}\right) \\ & - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6\right) \\ & - \left(C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6\right) \\ & - \left(C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6\right) \\ & - \left(C_{i,45}H_4H_5 + C_{i,46}H_4H_6\right) \\ & - \left(C_{i,56}H_5H_6\right) \\ & - \left(C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + C_{i,333}H_3^{\,3} + C_{i,444}H_4^{\,3} + C_{i,555}H_5^{\,3} + C_{i,666}H_6^{\,3}\right) \end{aligned}$$

A.3.4 Second order, 6 component equation with 84 coefficients

$$
\begin{aligned}
H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6 \\
& - \left(C_{i,1}|H_1| + C_{i,2}|H_2| + C_{i,3}|H_3| + C_{i,4}|H_4| + C_{i,5}|H_5| + C_{i,6}|H_6|\right) \\
& - \left(C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2}\right) \\
& - \left(C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + C_{i,3|3|}H_3|H_3| + C_{i,4|4|}H_4|H_4| + C_{i,5|5|}H_5|H_5| + C_{i,6|6|}H_6|H_6|\right) \\
& - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6\right) \\
& - \left(C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6\right) \\
& - \left(C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6\right) \\
& - \left(C_{i,45}H_4H_5 + C_{i,46}H_4H_6\right) \\
& - \left(C_{i,56}H_5H_6\right) \\
& - \left(C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + C_{i,|14|}|H_1H_4| + C_{i,|15|}|H_1H_5| + C_{i,|16|}|H_1H_6|\right) \\
& - \left(C_{i,|23|}|H_2H_3| + C_{i,|24|}|H_2H_4| + C_{i,|25|}|H_2H_5| + C_{i,|26|}|H_2H_6|\right) \\
& - \left(C_{i,|34|}|H_3H_4| + C_{i,|35|}|H_3H_5| + C_{i,|36|}|H_3H_6|\right) \\
& - \left(C_{i,|45|}|H_4H_5| + C_{i,|46|}|H_4H_6|\right) \\
& - \left(C_{i,|56|}|H_5H_6|\right) \\
& - \left(C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + C_{i,1|4|}H_1|H_4| + C_{i,1|5|}H_1|H_5| + C_{i,1|6|}H_1|H_6|\right) \\
& - \left(C_{i,2|3|}H_2|H_3| + C_{i,2|4|}H_2|H_4| + C_{i,2|5|}H_2|H_5| + C_{i,2|6|}H_2|H_6|\right) \\
& - \left(C_{i,3|4|}H_3|H_4| + C_{i,3|5|}H_3|H_5| + C_{i,3|6|}H_3|H_6|\right) \\
& - \left(C_{i,4|5|}H_4|H_5| + C_{i,4|6|}H_4|H_6|\right) \\
& - \left(C_{i,5|6|}H_5|H_6|\right) \\
& - \left(C_{i,|1|2}|H_1|H_2 + C_{i,|1|3}|H_1|H_3 + C_{i,|1|4}|H_1|H_4 + C_{i,|1|5}|H_1|H_5 + C_{i,|1|6}|H_1|H_6\right) \\
& - \left(C_{i,|2|3}|H_2|H_3 + C_{i,|2|4}|H_2|H_4 + C_{i,|2|5}|H_2|H_5 + C_{i,|2|6}|H_2|H_6\right) \\
& - \left(C_{i,|3|4}|H_3|H_4 + C_{i,|3|5}|H_3|H_5 + C_{i,|3|6}|H_3|H_6\right) \\
& - \left(C_{i,|4|5}|H_4|H_5 + C_{i,|4|6}|H_4|H_6\right) \\
& - \left(C_{i,|5|6}|H_5|H_6\right)
\end{aligned}
$$

A.3.5 Third order, 6 component equation with 96 coefficients

$$
\begin{aligned}
H_i = {} & C_{i,1R}R_1 + C_{i,2R}R_2 + C_{i,3R}R_3 + C_{i,4R}R_4 + C_{i,5R}R_5 + C_{i,6R}R_6 \\
& - \left(C_{i,1}|H_1| + C_{i,2}|H_2| + C_{i,3}|H_3| + C_{i,4}|H_4| + C_{i,5}|H_5| + C_{i,6}|H_6|\right) \\
& - \left(C_{i,11}H_1^{\,2} + C_{i,22}H_2^{\,2} + C_{i,33}H_3^{\,2} + C_{i,44}H_4^{\,2} + C_{i,55}H_5^{\,2} + C_{i,66}H_6^{\,2}\right) \\
& - \left(C_{i,1|1|}H_1|H_1| + C_{i,2|2|}H_2|H_2| + C_{i,3|3|}H_3|H_3| + C_{i,4|4|}H_4|H_4| + C_{i,5|5|}H_5|H_5| + C_{i,6|6|}H_6|H_6|\right) \\
& - \left(C_{i,12}H_1H_2 + C_{i,13}H_1H_3 + C_{i,14}H_1H_4 + C_{i,15}H_1H_5 + C_{i,16}H_1H_6\right) \\
& - \left(C_{i,23}H_2H_3 + C_{i,24}H_2H_4 + C_{i,25}H_2H_5 + C_{i,26}H_2H_6\right) \\
& - \left(C_{i,34}H_3H_4 + C_{i,35}H_3H_5 + C_{i,36}H_3H_6\right) \\
& - \left(C_{i,45}H_4H_5 + C_{i,46}H_4H_6\right) \\
& - \left(C_{i,56}H_5H_6\right) \\
& - \left(C_{i,|12|}|H_1H_2| + C_{i,|13|}|H_1H_3| + C_{i,|14|}|H_1H_4| + C_{i,|15|}|H_1H_5| + C_{i,|16|}|H_1H_6|\right) \\
& - \left(C_{i,|23|}|H_2H_3| + C_{i,|24|}|H_2H_4| + C_{i,|25|}|H_2H_5| + C_{i,|26|}|H_2H_6|\right) \\
& - \left(C_{i,|34|}|H_3H_4| + C_{i,|35|}|H_3H_5| + C_{i,|36|}|H_3H_6|\right) \\
& - \left(C_{i,|45|}|H_4H_5| + C_{i,|46|}|H_4H_6|\right) \\
& - \left(C_{i,|56|}|H_5H_6|\right) \\
& - \left(C_{i,1|2|}H_1|H_2| + C_{i,1|3|}H_1|H_3| + C_{i,1|4|}H_1|H_4| + C_{i,1|5|}H_1|H_5| + C_{i,1|6|}H_1|H_6|\right) \\
& - \left(C_{i,2|3|}H_2|H_3| + C_{i,2|4|}H_2|H_4| + C_{i,2|5|}H_2|H_5| + C_{i,2|6|}H_2|H_6|\right) \\
& - \left(C_{i,3|4|}H_3|H_4| + C_{i,3|5|}H_3|H_5| + C_{i,3|6|}H_3|H_6|\right) \\
& - \left(C_{i,4|5|}H_4|H_5| + C_{i,4|6|}H_4|H_6|\right) \\
& - \left(C_{i,5|6|}H_5|H_6|\right) \\
& - \left(C_{i,|1|2}|H_1|H_2 + C_{i,|1|3}|H_1|H_3 + C_{i,|1|4}|H_1|H_4 + C_{i,|1|5}|H_1|H_5 + C_{i,|1|6}|H_1|H_6\right) \\
& - \left(C_{i,|2|3}|H_2|H_3 + C_{i,|2|4}|H_2|H_4 + C_{i,|2|5}|H_2|H_5 + C_{i,|2|6}|H_2|H_6\right) \\
& - \left(C_{i,|3|4}|H_3|H_4 + C_{i,|3|5}|H_3|H_5 + C_{i,|3|6}|H_3|H_6\right) \\
& - \left(C_{i,|4|5}|H_4|H_5 + C_{i,|4|6}|H_4|H_6\right) \\
& - \left(C_{i,|5|6}|H_5|H_6\right) \\
& - \left(C_{i,111}H_1^{\,3} + C_{i,222}H_2^{\,3} + C_{i,333}H_3^{\,3} + C_{i,444}H_4^{\,3} + C_{i,555}H_5^{\,3} + C_{i,666}H_6^{\,3}\right) \\
& - \left(C_{i,|111|}\left|H_1^{\,3}\right| + C_{i,|222|}\left|H_2^{\,3}\right| + C_{i,|333|}\left|H_3^{\,3}\right| + C_{i,|444|}\left|H_4^{\,3}\right| + C_{i,|555|}\left|H_5^{\,3}\right| + C_{i,|666|}\left|H_6^{\,3}\right|\right)
\end{aligned}
$$

Appendix B Program Input / Output Listings

B.1 Data Input File Sample – Subset of the 1886 data points

```
#
#          APPLIED LOADS TO BALANCE, N or N/m                    BRIDGE OUTPUT, mV
#            [X, Y, Z, l, m, n]                                    [X, Y, Z, l, m, n]
    -500.3662   -3.1758  -32.431   -0.0144   -4.9714    2.871   -4.7771   -0.0181   -0.0963   -0.0044   -0.1922    0.2241
   -1001.0657   -2.9842  -30.0435  -0.0113   -4.9785    2.872   -9.5667   -0.0231   -0.0825   -0.0004   -0.1918    0.235
    -750.012    -2.7484  -29.963   -0.0236   -4.9735    2.8914  -7.1702   -0.0179   -0.0836   -0.0034   -0.1927    0.2309
    -249.562    -3.1836  -31.4643  -0.0229   -4.9933    2.8945  -2.386    -0.0231   -0.0971   -0.0065   -0.191     0.2199
     243.8836   -3.2036  -28.8638  -0.0346   -4.9552    2.8587   2.3317   -0.0228   -0.0967   -0.0106   -0.1917    0.2068
     733.1924   -3.0203  -29.563   -0.0198   -4.9543    2.8782   7.016    -0.0195   -0.109    -0.0111   -0.1918    0.1962
     978.1721   -3.0231  -31.5958  -0.0129   -4.9595    2.8908   9.3583   -0.0176   -0.1204   -0.0138   -0.1958    0.1908
     487.9826   -3.4692  -32.1982  -0.0149   -4.9473    2.8845   4.6809   -0.0191   -0.1126   -0.0145   -0.1919    0.2025
      17.6682   -3.0932  -30.1759  -0.014    -4.9772    2.9038   0.1831   -0.0193   -0.0988   -0.012    -0.1939    0.212
     -17.6209  -1960.2437  -28.6609  -0.0678   -4.9751    2.7947   0.1743  -13.2643   -0.224     0.1071   -0.1816    0.0253
    -500.16    -1960.0059  -30.0459  -0.0558   -4.9763    2.86    -4.7771  -13.2602   -0.2191    0.1116   -0.1797    0.0458
   -1000.6539  -1959.7125  -30.8096  -0.0455   -4.9776    2.8914  -9.5691  -13.2572   -0.2138    0.1116   -0.1797    0.0621
    -750.0955  -1960.1892  -31.5204  -0.0697   -5.0049    2.9198  -7.1734  -13.2609   -0.2186    0.1108   -0.1819    0.0547
     -248.988  -1960.1771  -32.0048  -0.0588   -4.9811    2.8534  -2.388   -13.2606   -0.2274    0.1072   -0.1836    0.0357
     243.7544  -1959.8779  -32.301   -0.0457   -5.0088    2.8542   2.3282  -13.2628   -0.2374    0.1074   -0.1831    0.0175
     733.5828  -1960.1428  -30.5971  -0.0493   -4.9627    2.8588   7.0066  -13.2674   -0.2406    0.1074   -0.1781   -0.0031
     978.5089  -1960.7428  -32.1055  -0.0511   -4.9711    2.8518   9.3478  -13.2707   -0.2508    0.1073   -0.1784   -0.0111
     488.4377  -1959.8184  -31.731   -0.0596   -4.9657    2.8543   4.6703  -13.2666   -0.2412    0.1078   -0.1781    0.0028
      18.1498  -1960.2415  -31.0338  -0.0497   -4.9768    2.8443   0.1777  -13.2655   -0.2309    0.1109   -0.1834    0.0235
      17.9252  -1177.5552  -30.3641  -0.0415   -4.9663    2.8623   0.183   -7.9685   -0.1835    0.0883   -0.1882    0.0913
    -500.2687  -1177.6012  -31.4311  -0.0437   -4.9806    2.8795  -4.7708   -7.9635   -0.1756    0.0919   -0.185     0.1085
   -1000.6263  -1177.7139  -31.9753  -0.0738   -4.9632    2.8664  -9.5611   -7.9651   -0.1709    0.0917   -0.1852    0.1214
    -750.5641  -1177.5823  -31.1661  -0.0519   -4.9534    2.8734  -7.1658   -7.9633   -0.1714    0.0903   -0.1861    0.1151
     -249.388  -1177.4187  -31.3172  -0.0604   -4.9116    2.8554  -2.3831   -7.9679   -0.1788    0.0858   -0.1819    0.0998
     243.7241  -1177.9418  -30.0609  -0.0489   -4.987     2.8589   2.3342   -7.9701   -0.1869    0.0854   -0.1896    0.0835
     733.4962  -1178.2875  -31.9693  -0.034    -5.0179    2.857    7.0159   -7.9699   -0.1988    0.0818   -0.1852     0.07
     978.2535  -1178.0741  -31.1343  -0.0427   -4.985     2.86     9.3581   -7.9718   -0.2001    0.0806   -0.1855     0.06
     488.4358  -1177.8318  -30.6716  -0.053    -4.9992    2.844    4.6788   -7.971    -0.1916    0.0856   -0.19       0.077
      17.9254  -1177.907   -29.8247  -0.0433   -5.0138    2.8512   0.1833   -7.9674   -0.1807    0.0884   -0.1882     0.089
      17.9072  -394.3307   -31.3683  -0.01     -4.9972    2.8972   0.1845   -2.6773   -0.134     0.03     -0.1943    0.1708
     -500.52   -394.7624   -31.5946  -0.024    -4.9969    2.8852  -4.7736   -2.6818   -0.1259    0.0344   -0.192     0.1866
   -1001.0369  -394.5432   -29.3742  -0.0154   -4.9423    2.8896  -9.5641   -2.6808   -0.1136    0.0377   -0.1887    0.1986
    -750.5283  -394.9583   -31.0408  -0.0081   -5.0021    2.8775  -7.169    -2.6815   -0.1202    0.035    -0.1917    0.1927
    -249.6237  -394.4568   -30.4081  -0.0121   -4.9327    2.8993  -2.3856   -2.6818   -0.1274    0.0308   -0.1928    0.1794
     243.5932  -395.1572   -31.4413  -0.0067   -4.9409    2.8675   2.3343   -2.6819   -0.1397    0.0279   -0.1934    0.1652
     733.1033  -395.0909   -31.4051  0.0065    -4.9736    2.864    7.0155   -2.681    -0.1446    0.0245   -0.1923    0.1526
     978.2639  -395.3293   -32.605   -0.0026   -4.9716    2.8653   9.3614   -2.6804   -0.1526    0.023    -0.1884    0.1475
      487.921  -395.3039   -30.9188  0.0106    -4.9907    2.8422   4.68     -2.6804   -0.1421    0.0257   -0.1924    0.1593
      17.9873  -394.8606   -30.1881  0.0007    -5.0115    2.8895   0.1862   -2.6814   -0.1304    0.0303   -0.196     0.1731
      17.6976  392.8755    -31.7996  0.0048    -4.9883    2.8907   0.1757   2.6568    -0.0742    -0.0254  -0.1996    0.2463
     -500.6028  393.1338   -29.5834  -0.0013   -4.9937    2.8783  -4.7823   2.6574    -0.0569    -0.0244  -0.1955    0.2581
    -1001.2208  392.5925   -30.7709  -0.0034   -5.0078    2.8787  -9.5734   2.6521    -0.0524    -0.0202  -0.1962    0.268
     -750.3299  392.7025   -29.3489  -0.005    -4.9668    2.8729  -7.1796   2.6541    -0.0528    -0.0204  -0.1964    0.2646
     -249.6333  392.7539   -30.0833  -0.0017   -4.844     2.913    -2.3914   2.6544    -0.0625    -0.023   -0.1885    0.2555
      243.9433  392.4188   -31.5702  -0.0014   -4.8858    2.8877   2.3268   2.6538    -0.0749    -0.0234  -0.1917    0.2398
      733.1606  392.7461   -29.0611  -0.0017   -4.8899    2.8779   7.0062   2.6571    -0.0759    -0.0305  -0.1879    0.2305
      977.8995   392.539   -30.8882            -4.8609    2.8536   9.3495   2.6538    -0.086     -0.0313  -0.1898    0.2251
```

B.2 Balance Design Load Range Input File

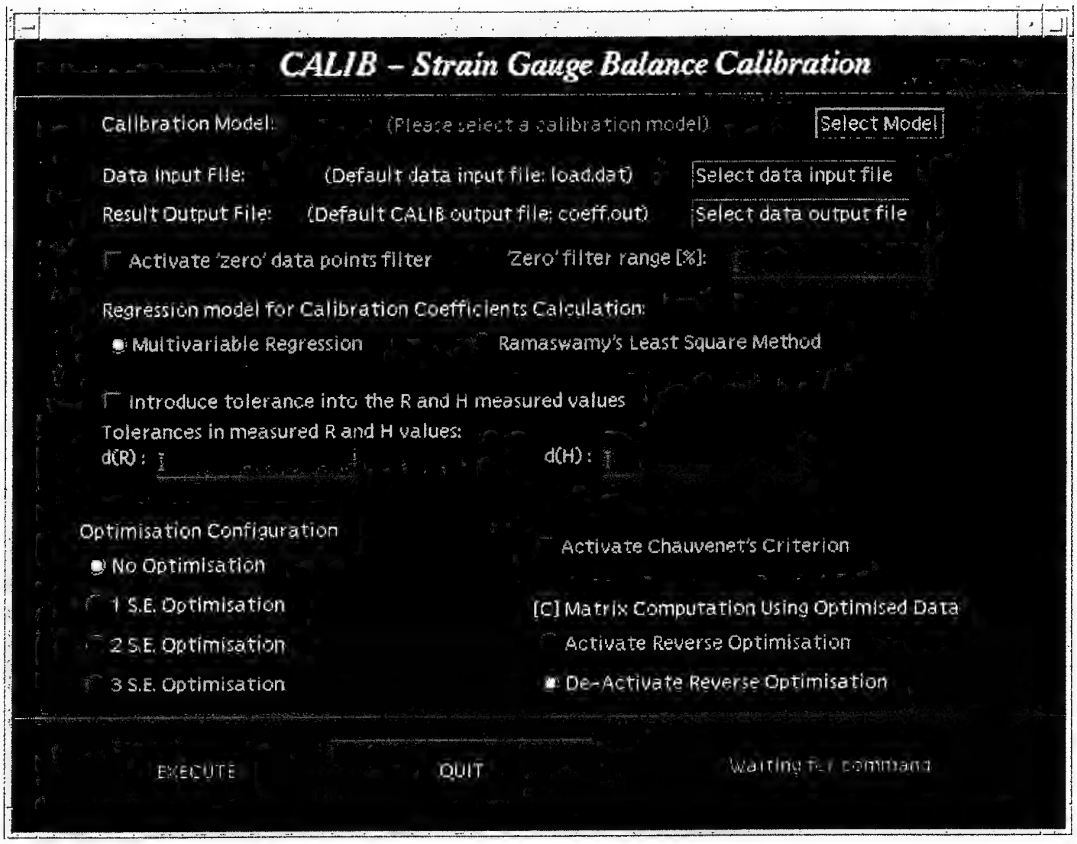
```
# ================================================================
# This file contains the balance loading range for the Aerotech
# balance
# ================================================================
# All unit are in N/Nm
#
# Axial (X)
  1000
#
# Side (Y)
  2000
#
# Lift (Z)
  5000
#
# Roll (L)
  100
#
# Pitch (M)
  300
#
# Yaw (N)
  150
```

B.3 *CALIB*'s Output File

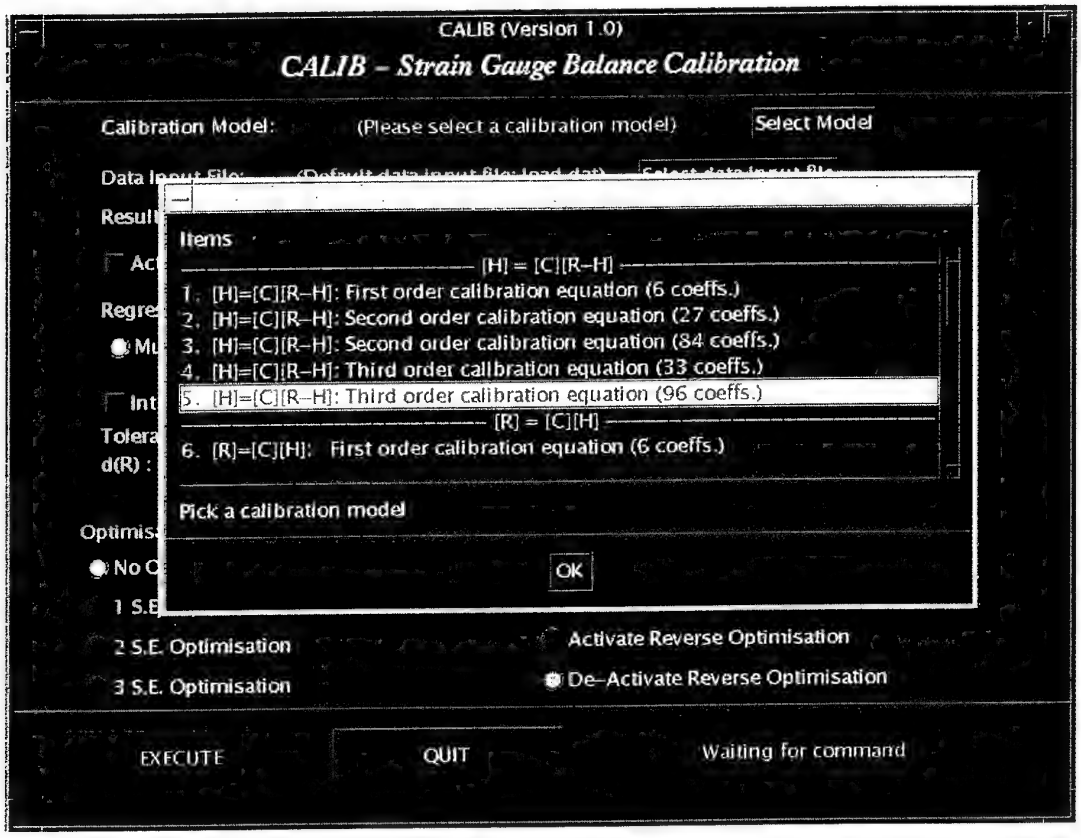
```
# ====================================================
# This data output file is generated by program: calib
# ====================================================

# Balance Calibration model: AMRL's first order calibration equation
# Model equation:
#               R = C1*HX + C2*HY + C3*HZ + C4*Hl + C5*Hm + C6*Hn

# No. of calibration coefficients: 6
# Mathematical regression model used: Multivariable regression

# Tolerance in the original [H] measurements: 0.000%
# Tolerance in the original [R] measurements: 0.000%

# 'Zero data point(s)' filter NOT ACTIVATED

# Optimisation for outlier(s) elimination: NOT ACTIVATED

# Chauvenet's Criterion for outlier(s) elimination: NOT ACTIVATED

# The calibration data input file contains 1886 data points.

# ------------------------------------------------
# Balance Calibration Coefficients (Transposed)
# ------------------------------------------------
#          HX              HY              HZ              Hl              Hm              Hn
C01   9.56131564e-03   1.54069253e-06  -1.71699722e-05  -3.71443729e-06   9.26901859e-07  -2.37977230e-05
C02  -1.11586112e-05   6.77280604e-03   7.10106126e-05  -3.18480774e-05  -4.44941489e-06   8.73411563e-05
C03  -3.55989620e-05   1.38857130e-05   3.24190447e-03   6.99404024e-05  -1.64403613e-05  -6.03888326e-06
C04   4.67655411e-04   1.03005363e-03  -5.75349267e-04   9.30475144e-02   1.03809403e-03  -1.35076731e-02
C05   6.46963845e-04   3.04742113e-06   2.50076814e-04   6.49532542e-04   3.97225368e-02   1.18080339e-04
C06   8.21138210e-04  -2.83188524e-04   1.00852295e-05   2.84751339e-04   8.74790649e-04   7.42214482e-02

# ------------------------------------------------------------
# Normalised Balance Calibration Coefficients (Transposed)
# ------------------------------------------------------------
#          HX              HY              HZ              Hl              Hm              Hn
C01   1.00000000e+00   2.27482157e-04  -5.29626097e-03  -3.99197907e-05   2.33344075e-05  -3.20631348e-04
C02  -1.16705813e-03   1.00000000e+00   2.19039806e-02  -3.42277573e-04  -1.12012355e-04   1.17676438e-03
C03  -3.72322841e-03   2.05021566e-03   1.00000000e+00   7.51663307e-04  -4.13879944e-04  -8.13630480e-05
C04   4.89111989e-02   1.52086687e-01  -1.77472616e-01   1.00000000e+00   2.61336288e-02  -1.81991505e-01
C05   6.76647304e-02   4.49949565e-04   7.71388596e-02   6.98065442e-03   1.00000000e+00   1.59091936e-03
C06   8.58812993e-02  -4.18125844e-02   3.11089656e-03   3.06027884e-03   2.20225273e-02   1.00000000e+00

# ------------------
# Sensitivity Matrix
# ------------------
#          HX              HY              HZ              Hl              Hm              Hn
C01   9.56131564e-03   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00
C02   0.00000000e+00   6.77280604e-03   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00
C03   0.00000000e+00   0.00000000e+00   3.24190447e-03   0.00000000e+00   0.00000000e+00   0.00000000e+00
C04   0.00000000e+00   0.00000000e+00   0.00000000e+00   9.30475144e-02   0.00000000e+00   0.00000000e+00
C05   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00   3.97225368e-02   0.00000000e+00
C06   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00   0.00000000e+00   7.42214482e-02


# ---------------
# Standard Errors
# ---------------
#          HX              HY              HZ              Hl              Hm              Hn
        0.43587058%     0.16990011%     0.13213621%     0.60054345%     0.07482476%     0.14332177%

# ----------------------------------
# Coefficient of multiple correlation
# ----------------------------------
#          HX              HY              HZ              Hl              Hm              Hn
        0.99993341      0.99998672      0.99999345      0.99992707      0.99999778      0.99999451
```

Appendix C Computer Program's Graphical User Interface (GUI)

Running CALIB provides the user with the following window:

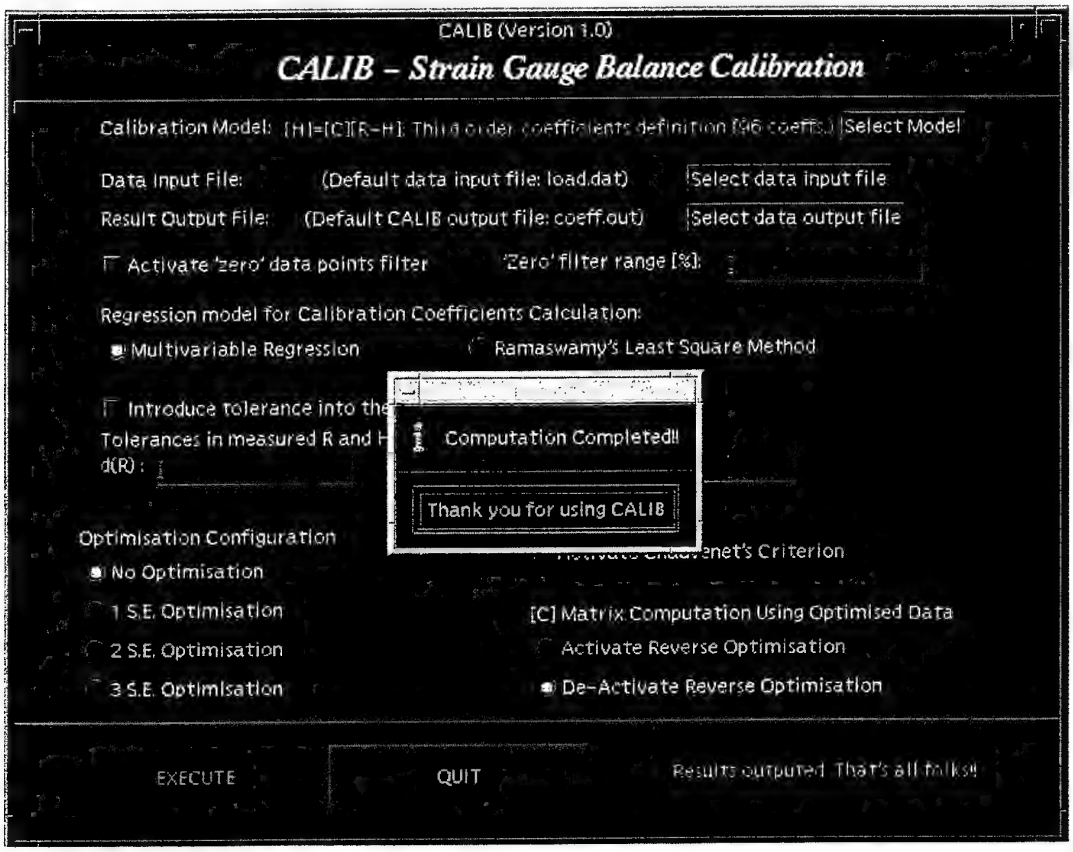


By clicking Select Model , a separate window containing all the available calibration equations will appear as shown below.



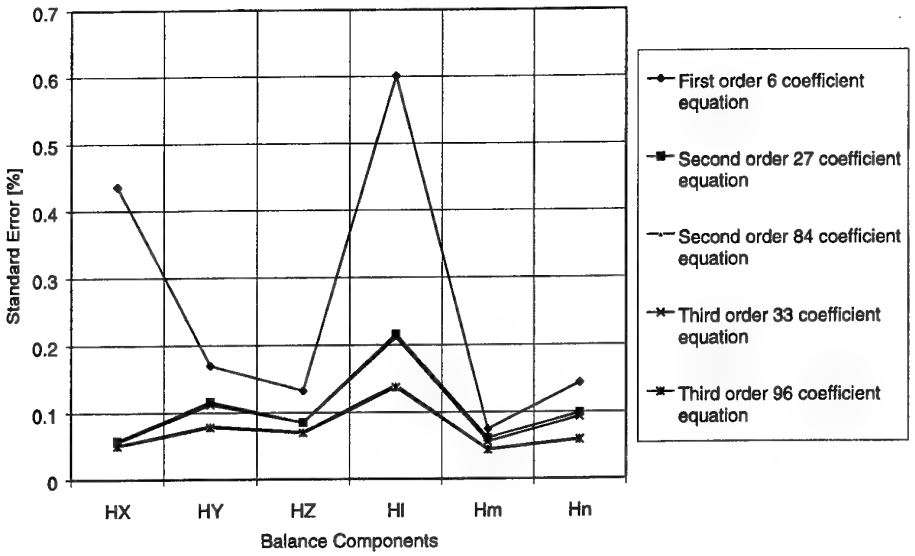
After all the selections have been made, the program will begin the analysis by clicking

After the program has successfully finished the computation, a separate window will appear on top of the original window, indicating the computation is completed.

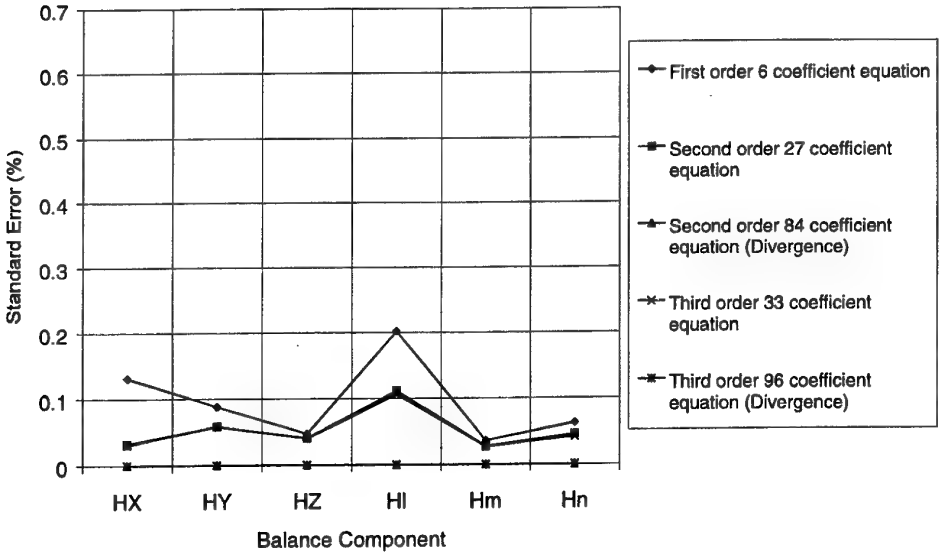


Appendix D Graphs

D.1 Calibration Model: [R]=[C][H] (1886 calibration data set)

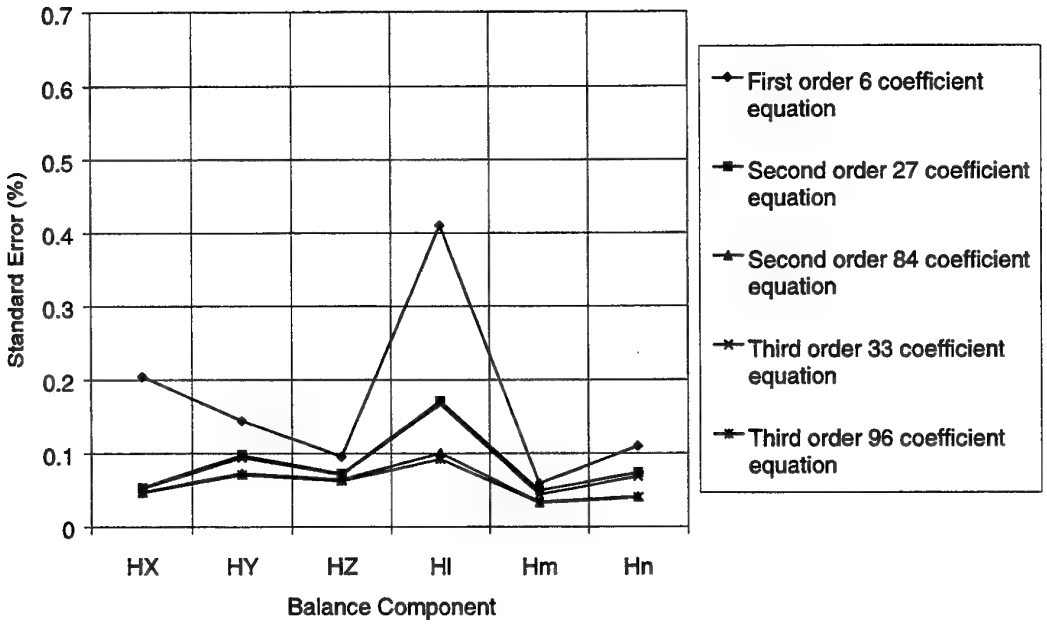


a. No optimisation

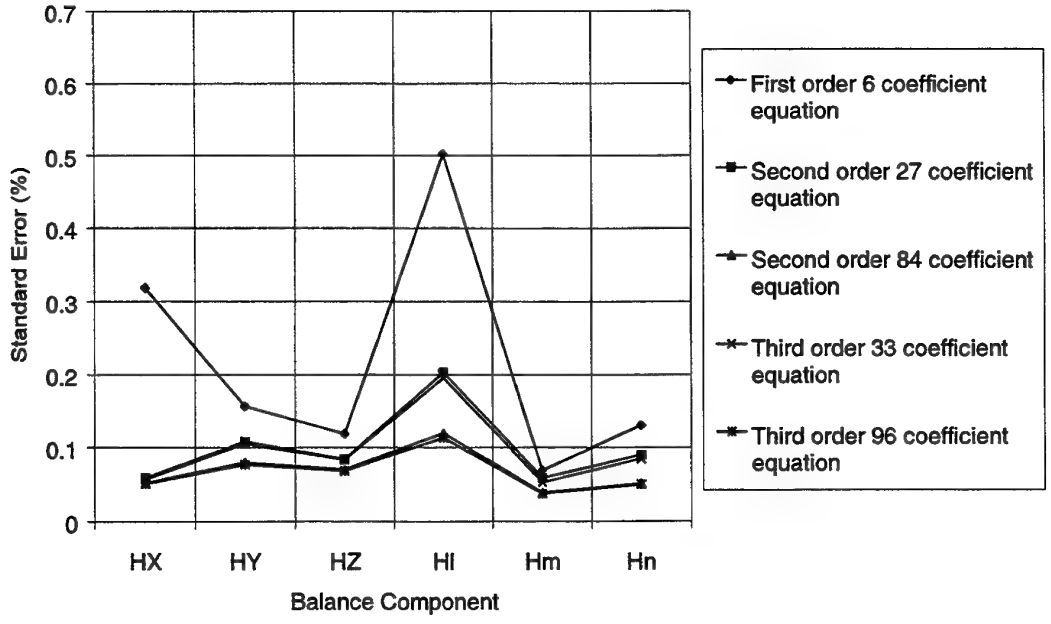


b. 1se optimisation

Figure 10. Balance Calibration Model [R]=[C][H] with 1886 data points

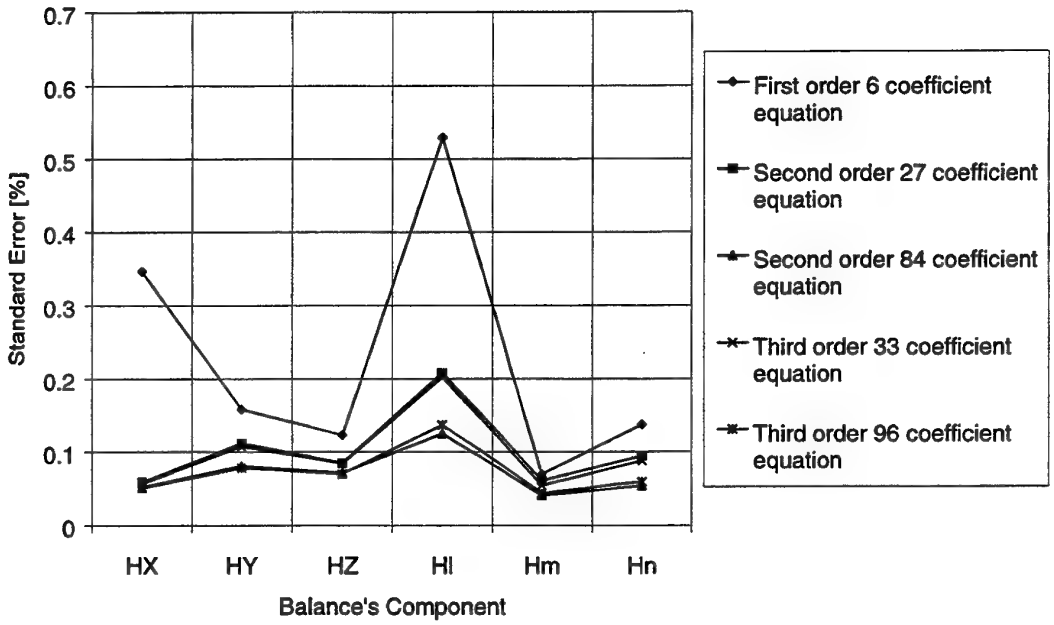


c. 2se optimisation

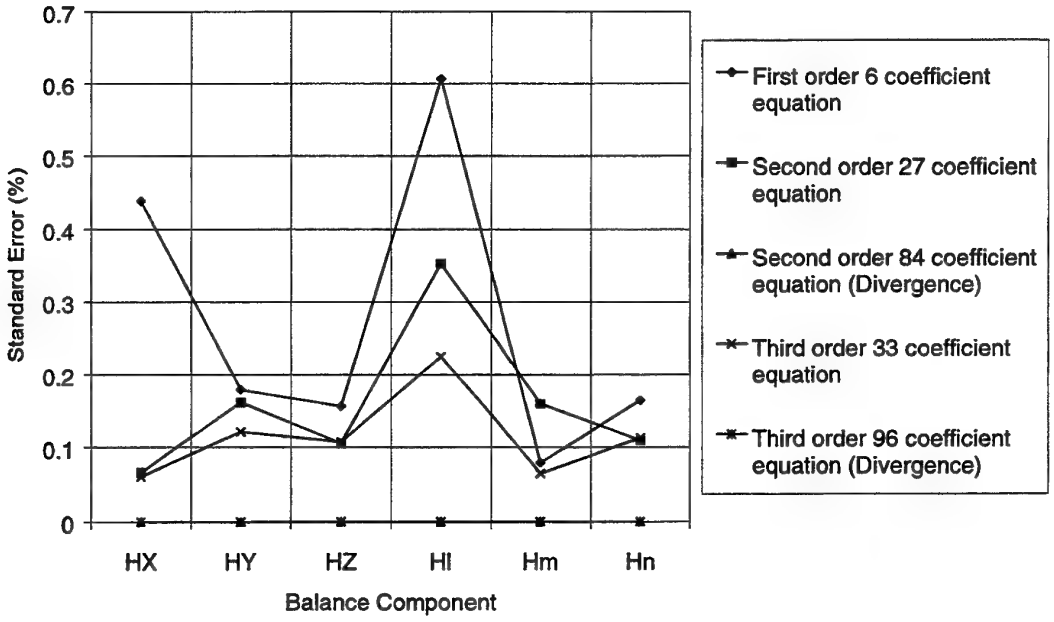


d. 3se optimisation

Figure 10. Balance Calibration Model [R]=[C][H] with 1886 data points (*cont'd*)

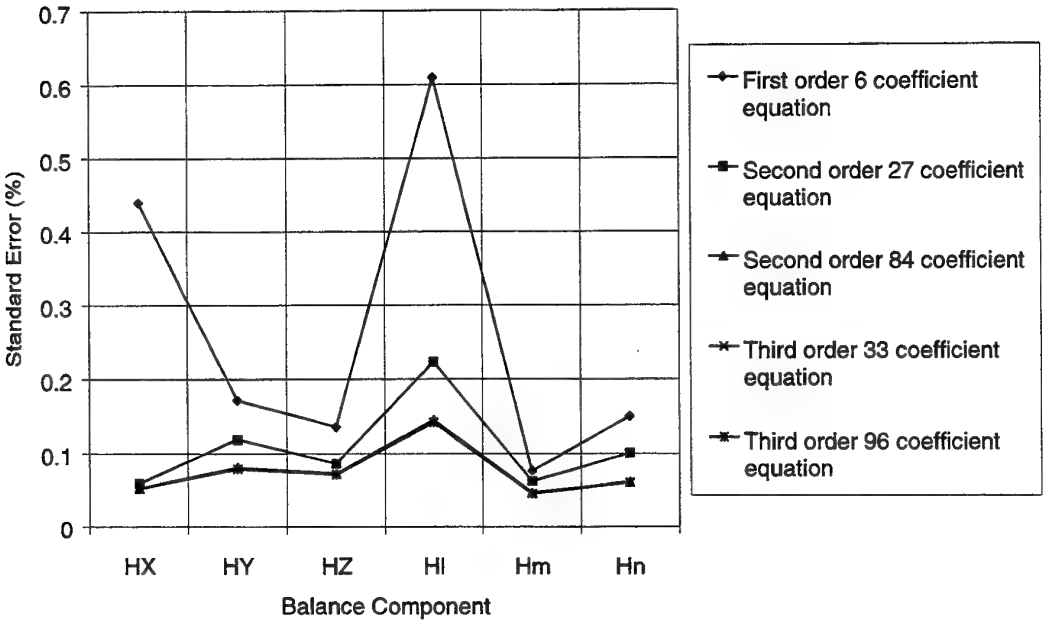


e. Chauvenet's Criterion optimisation

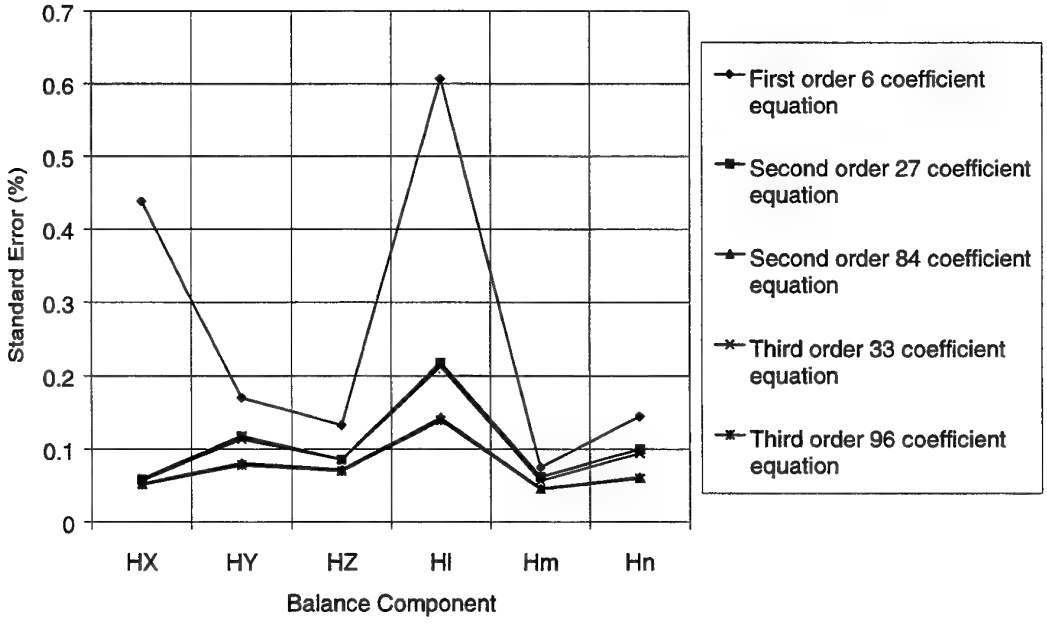


f. 1se optimisation with non-optimised calibration data

Figure 10. Balance Calibration Model [R]=[C][H] with 1886 data points (cont'd)

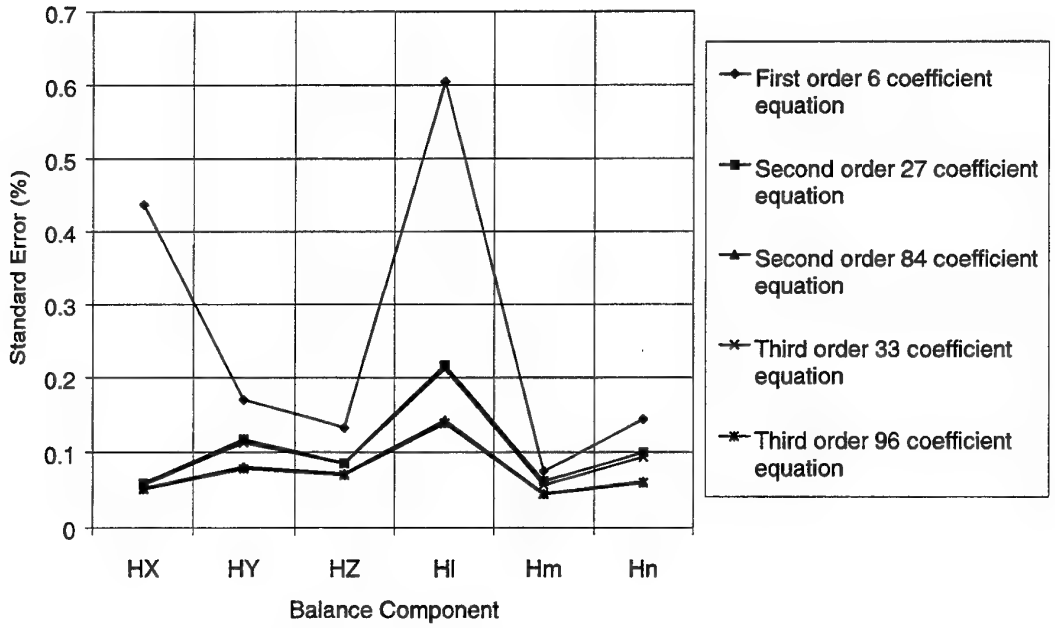


g. 2se optimisation with non-optimised calibration data



h. 3se optimisation with non-optimised calibration data

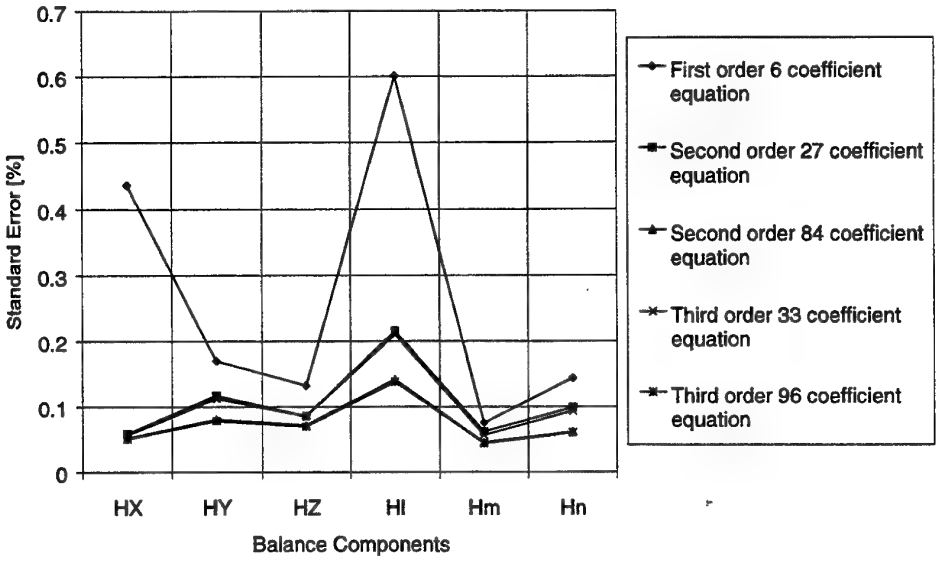
Figure 10. Balance Calibration Model [R]=[C][H] with 1886 data points (cont'd)



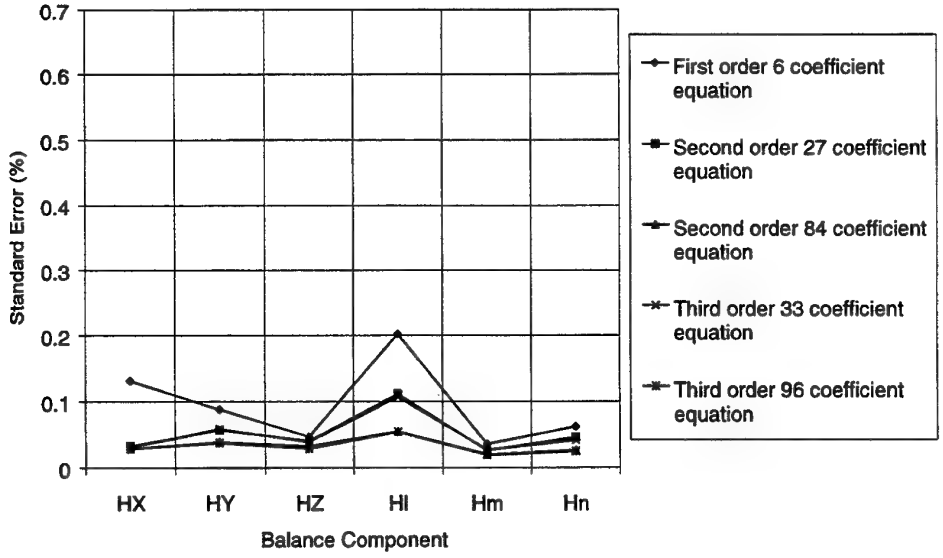
i. Chauvenet's Criterion with non-optimised calibration data

Figure 10. Balance Calibration Model [R]=[C][H] with 1886 data points (cont'd)

D.2 Calibration Model: [H]=[C][R] (1886 calibration data set)

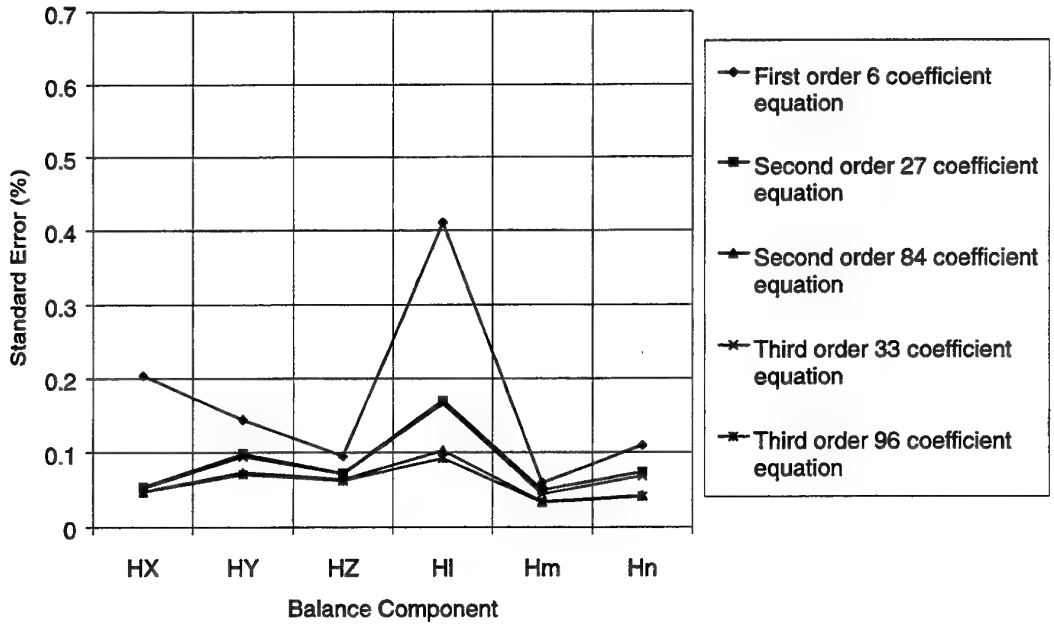


a. No optimisation

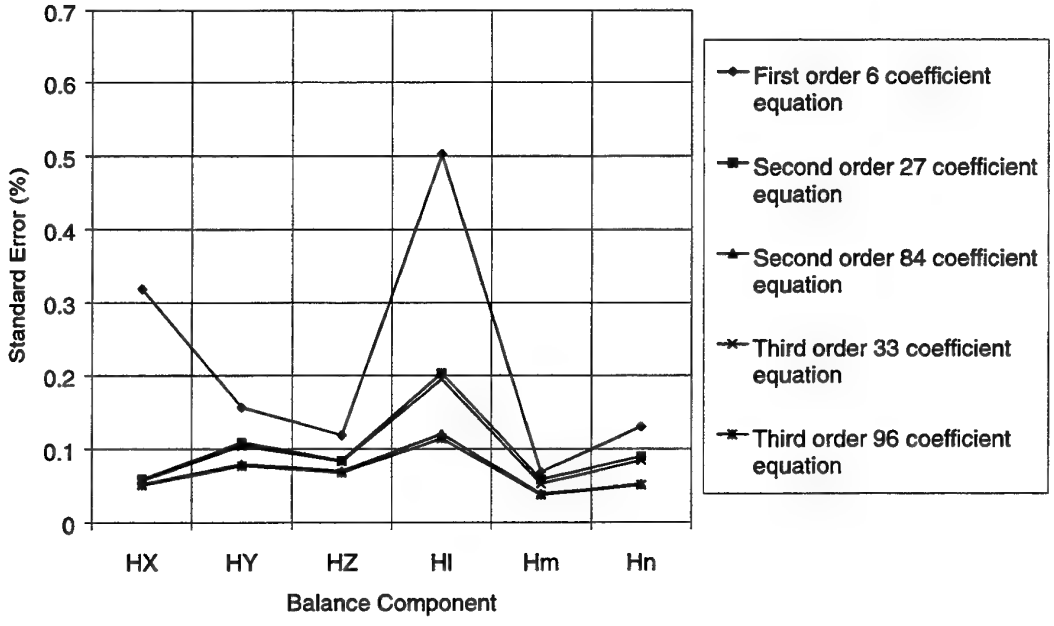


b. 1se optimisation

Figure 11. Balance Calibration Model, [H]=[C][R] with 1886 data points

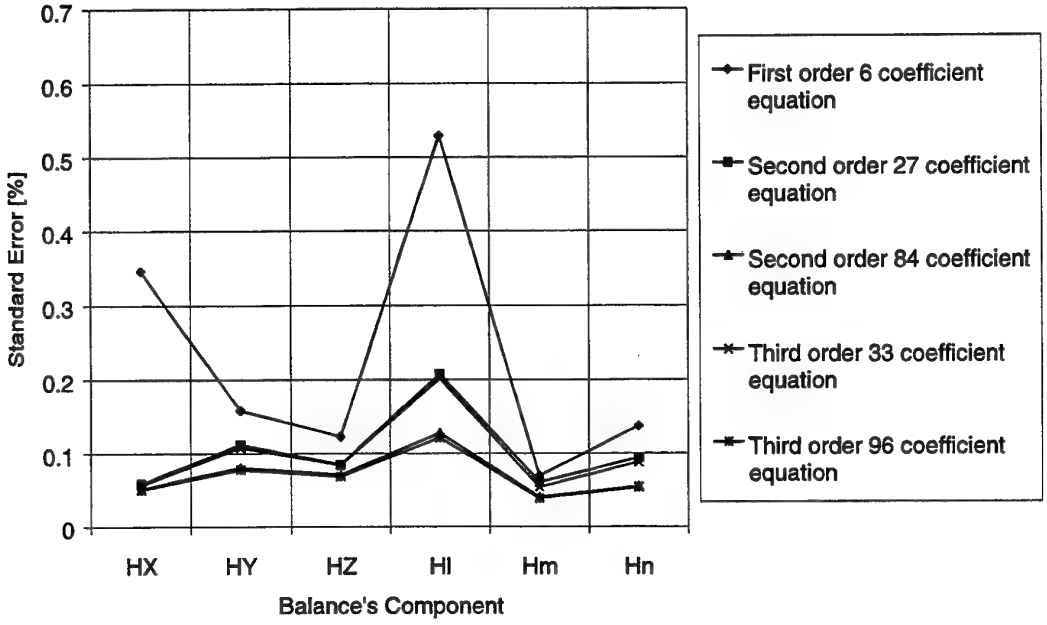


c. 2se optimisation

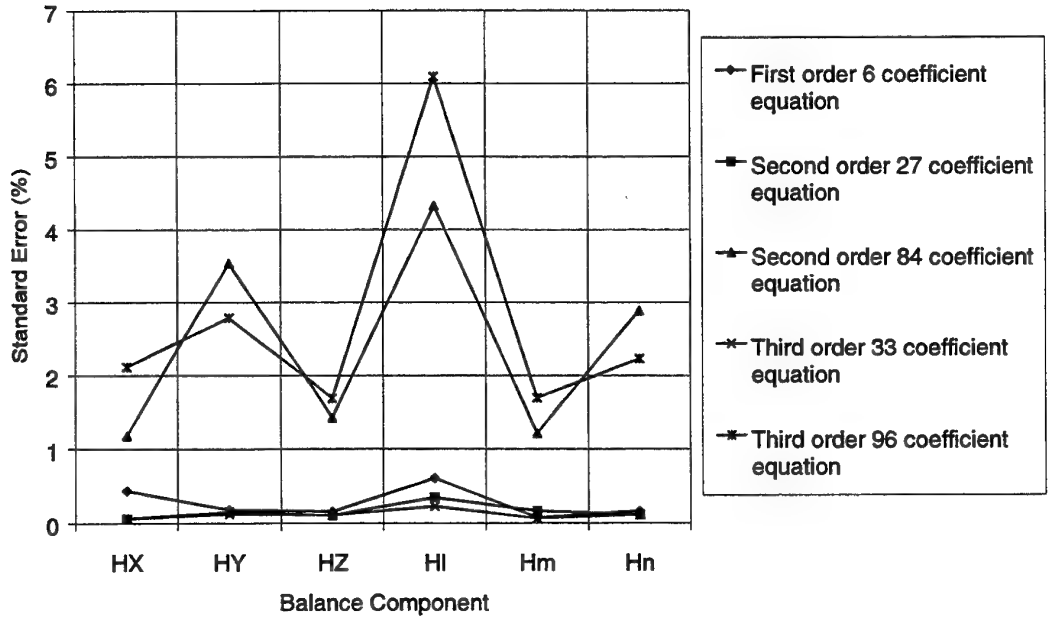


d. 3se optimisation

Figure 11. Balance Calibration Model, [H]=[C][R] with 1886 data points (cont'd)

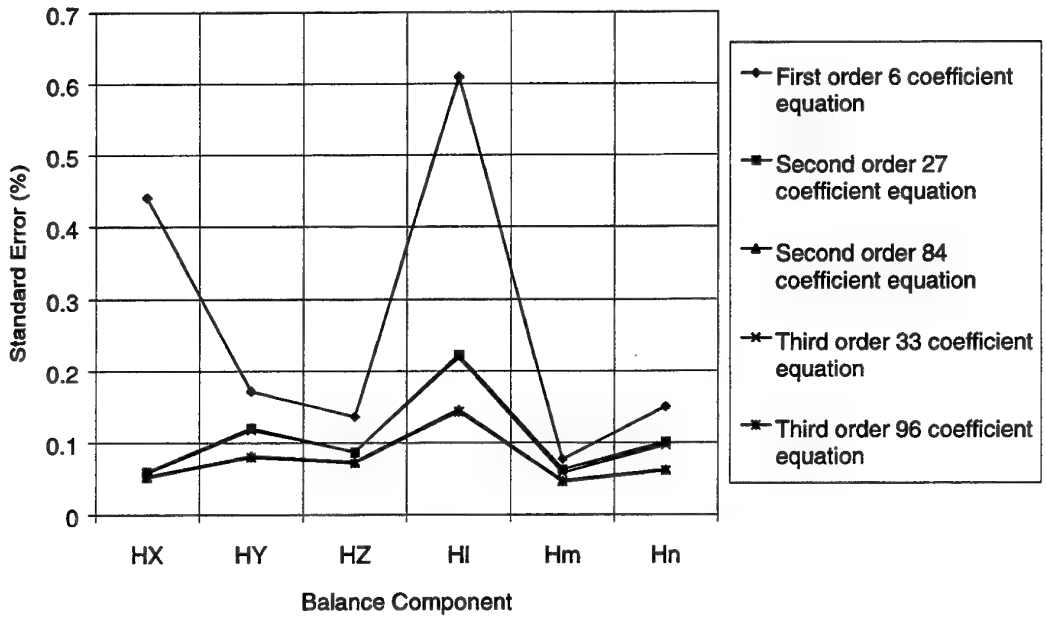


e. Chauvenet's Criterion optimisation

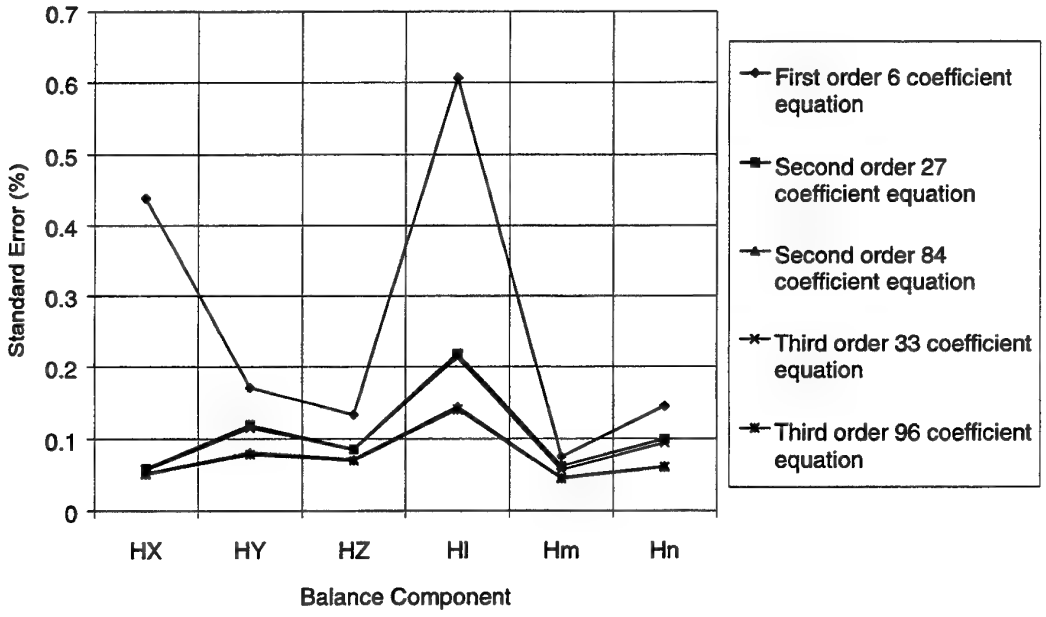


f. 1se optimisation with non-optimised calibration data

Figure 11. Balance Calibration Model, [H]=[C][R] with 1886 data points (cont'd)

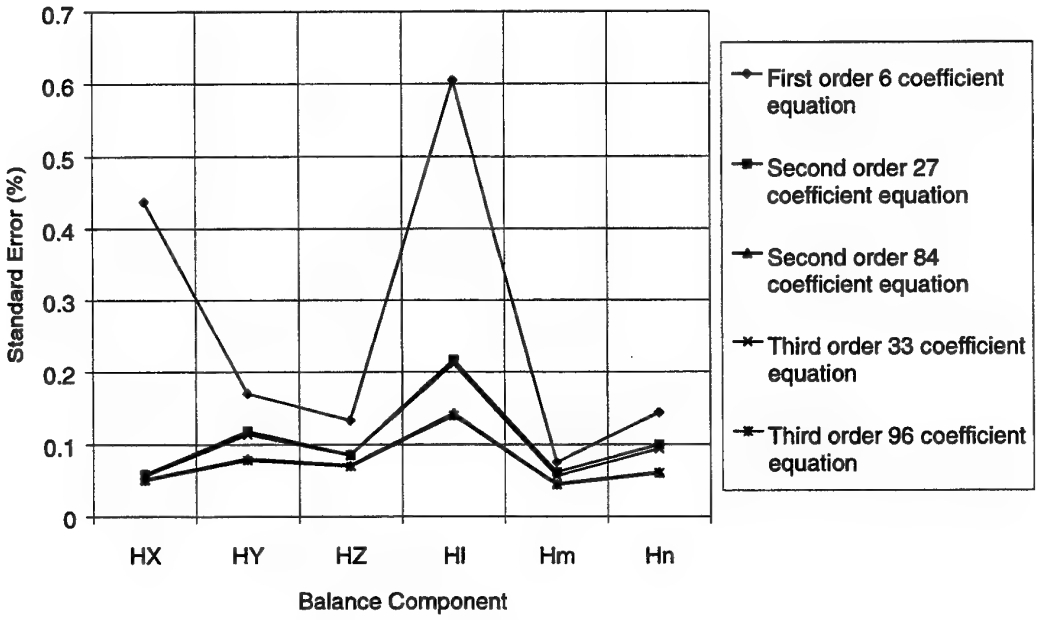


g. 2se optimisation with non-optimised calibration data



h. 3se optimisation with non-optimised calibration data

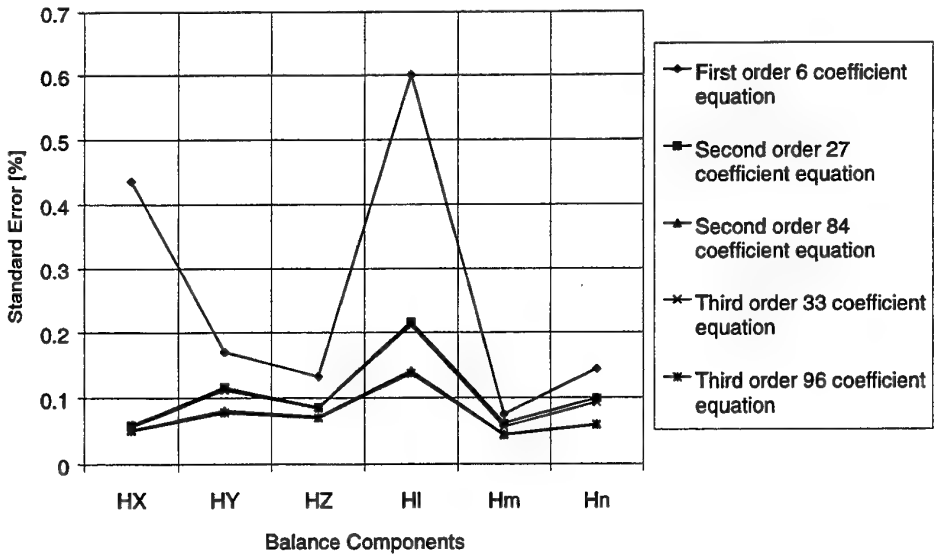
Figure 11. Balance Calibration Model, [H]=[C][R] with 1886 data points (cont'd)



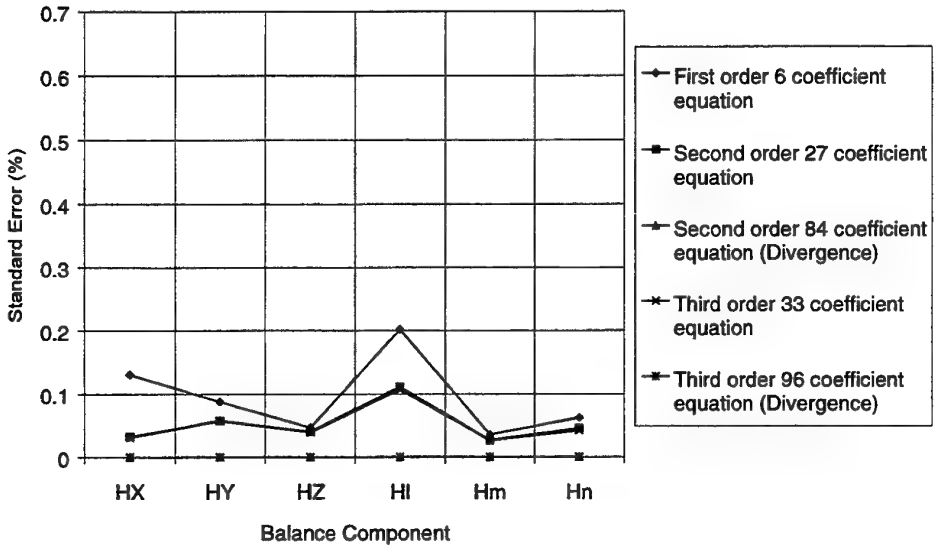
i. Chauvenet's Criterion with non-optimised calibration data

Figure 11. Balance Calibration Model, [H]=[C][R] with 1886 data points (cont'd)

D.3 Calibration Model: [H]=[C][R-H] (1886 calibration data set)

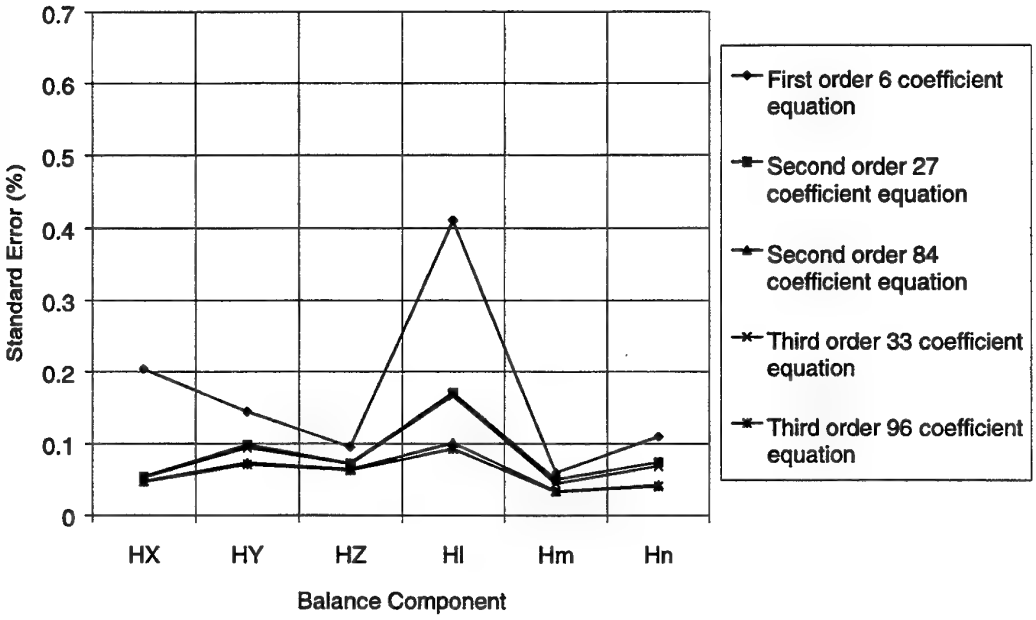


a. No optimisation

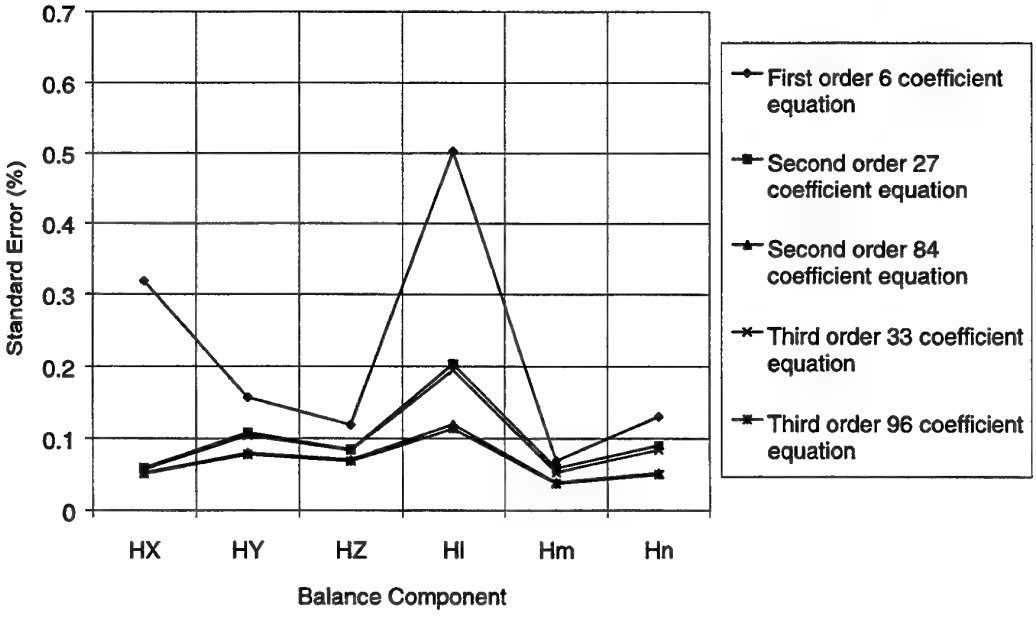


b. 1se optimisation

Figure 12. Balance Calibration Model, [H]=[C][R-H] with 1886 data points

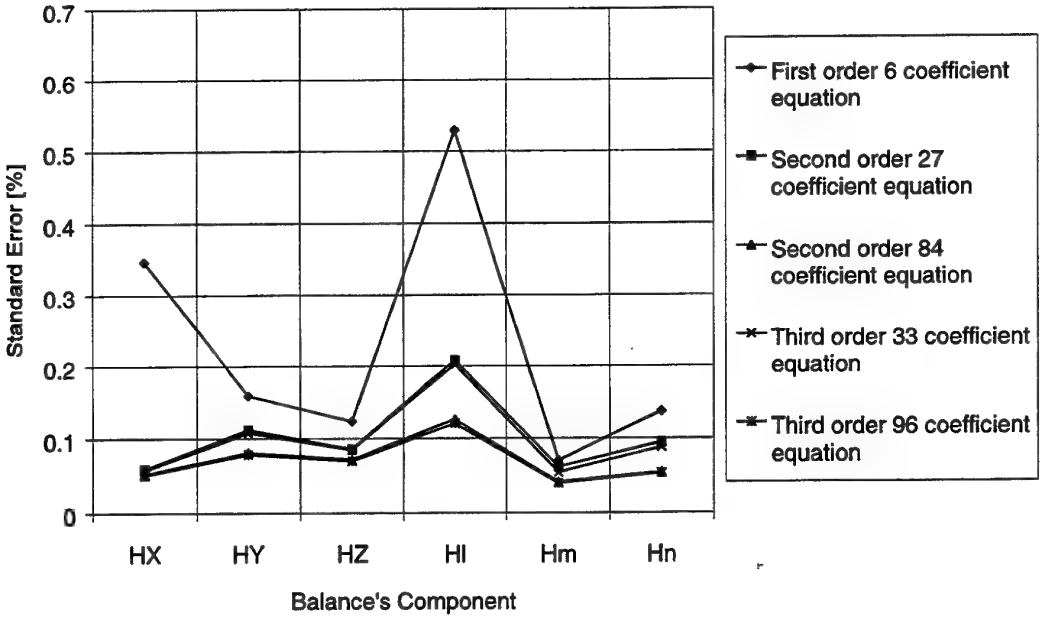


c. 2se optimisation

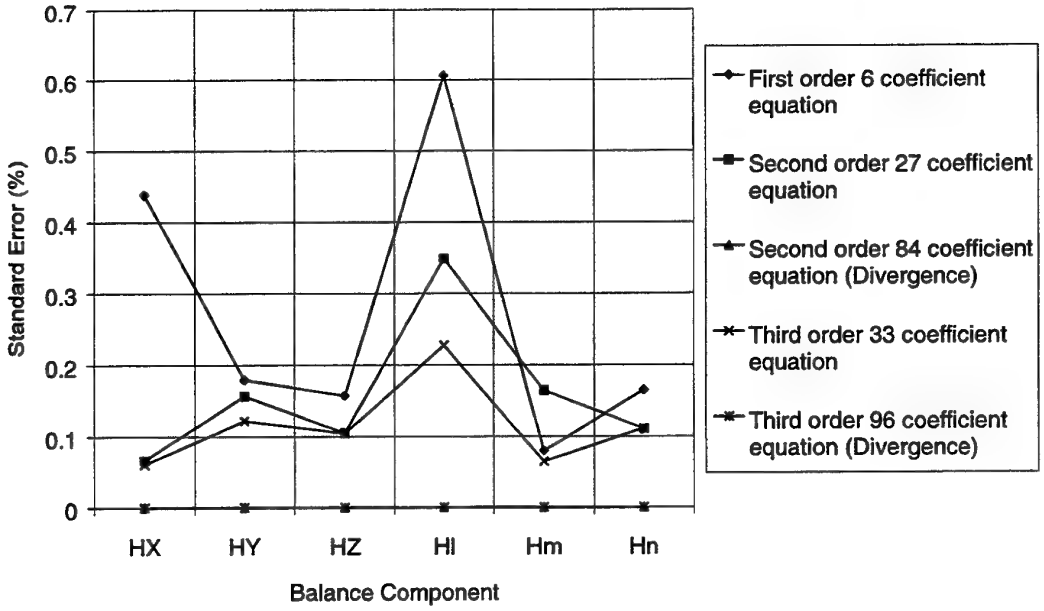


d. 3se optimisation

Figure 12. Balance Calibration Model, [H]=[C][R-H] with 1886 data points (cont'd)

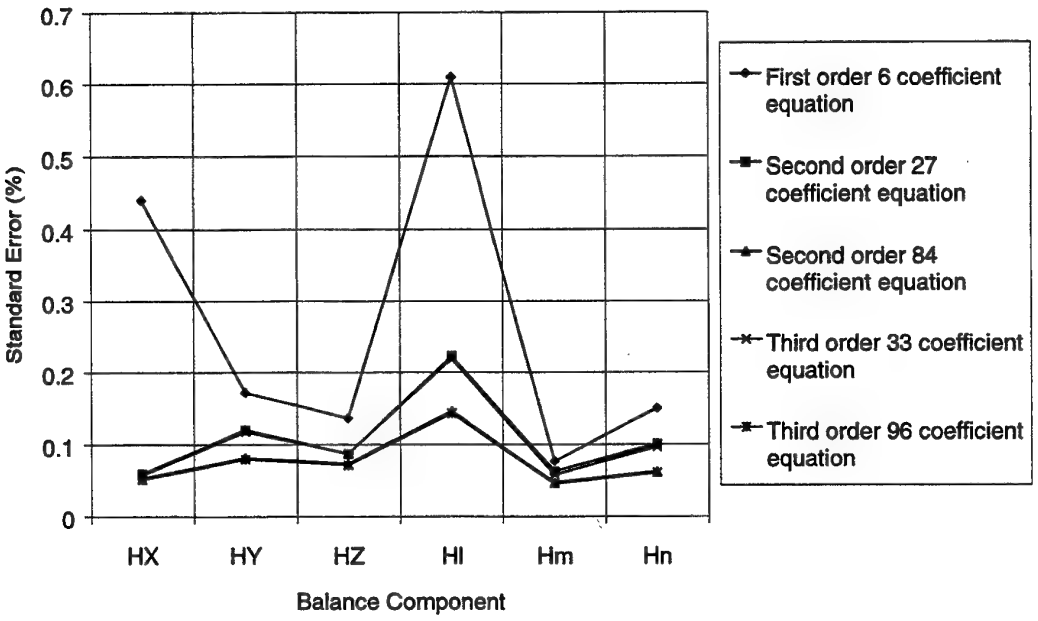


e. Chauvenet's Criterion optimisation

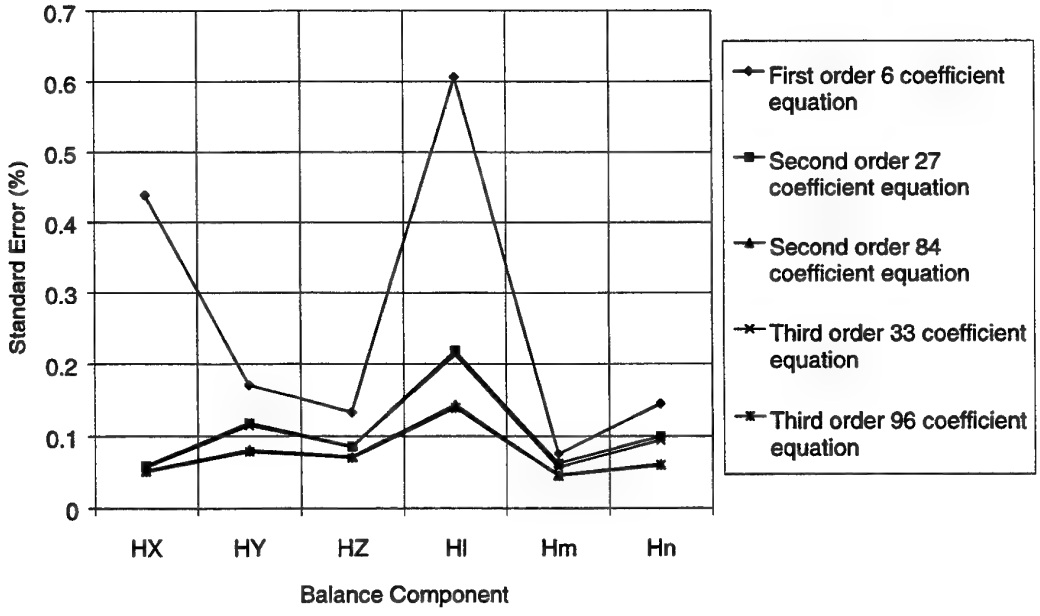


f. 1se optimisation with non-optimised calibration data

Figure 12. Balance Calibration Model, [H]=[C][R-H] with 1886 data points (cont'd)

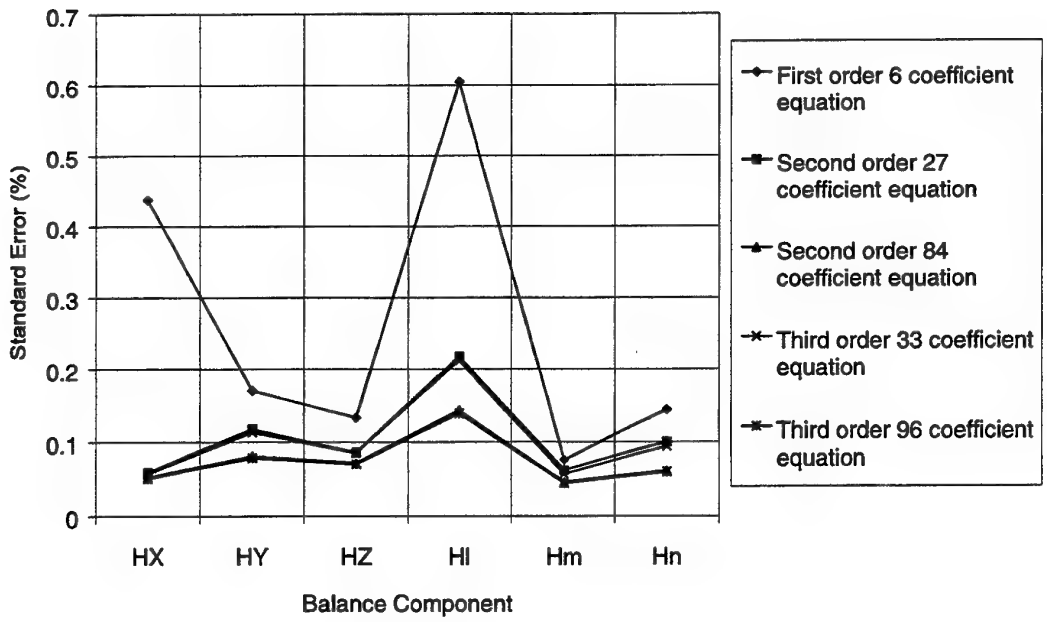


g. 2se optimisation with non-optimised calibration data



h. 3se optimisation with non-optimised calibration data

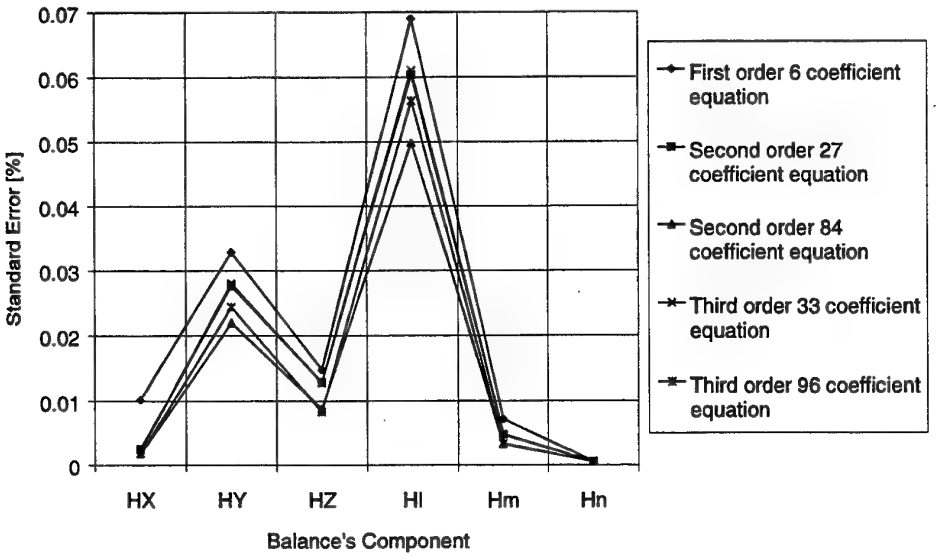
Figure 12. Balance Calibration Model, [H]=[C][R-H] with 1886 data points (cont'd)



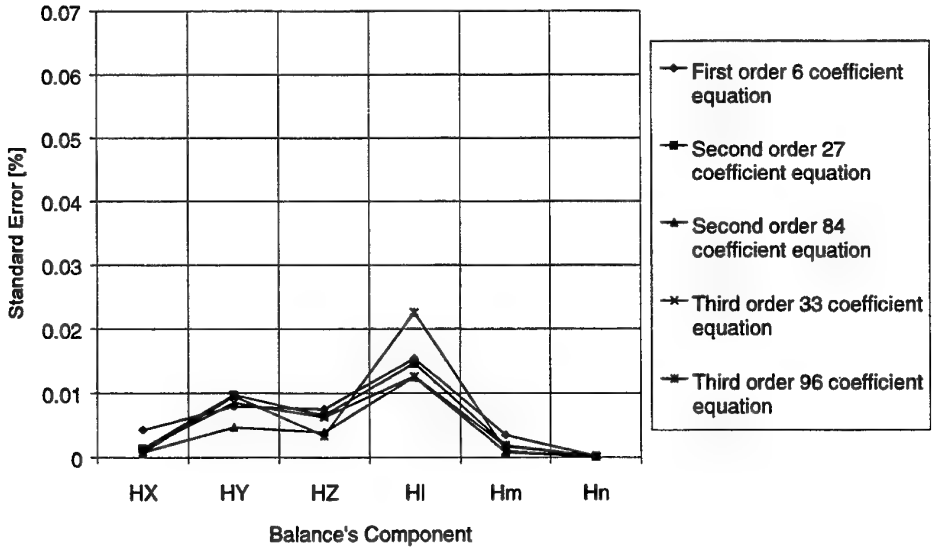
i. Chauvenet's Criterion with non-optimised calibration data

Figure 12. Balance Calibration Model, [H]=[C][R-H] with 1886 data points (cont'd)

D.4 Calibration Model: [R]=[C][H] (329 calibration data set)

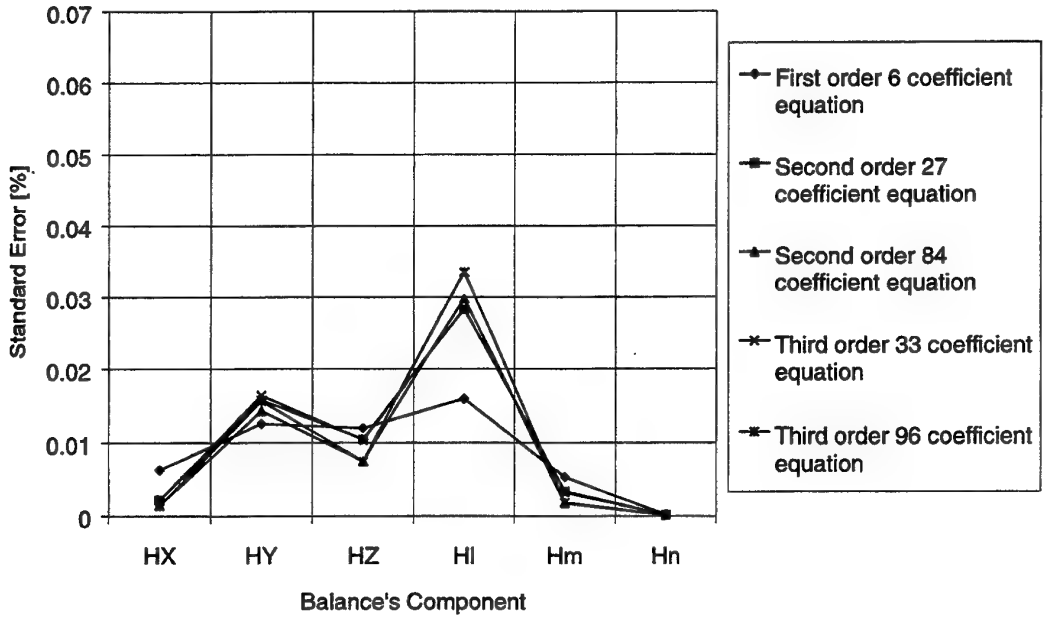


a. No optimisation

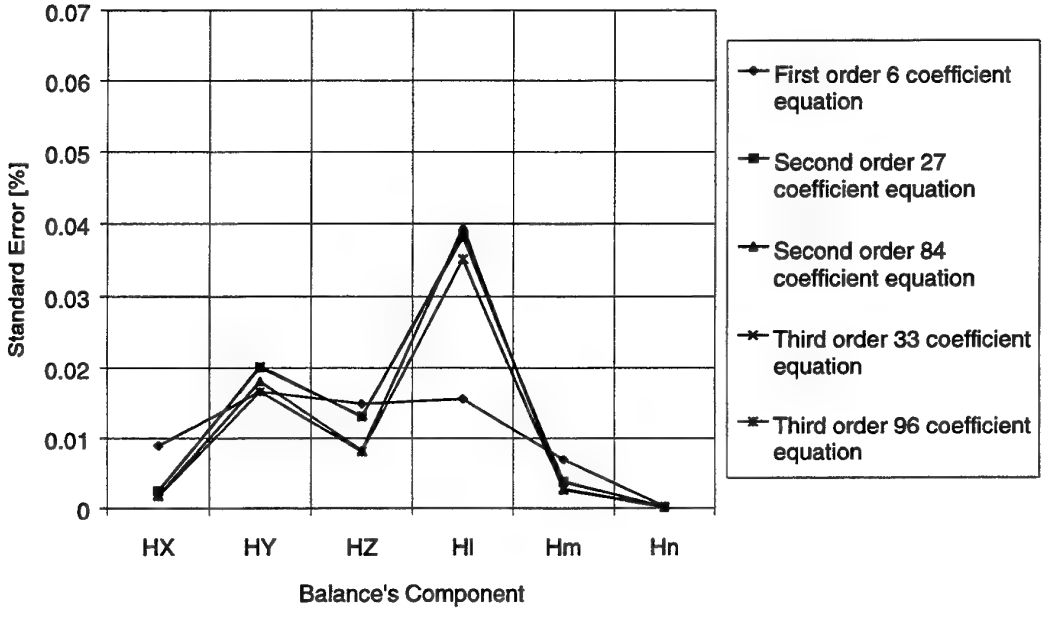


b. 1se Optimisation

Figure 13. Balance Calibration Model, [R]=[C][H] with 329 data points

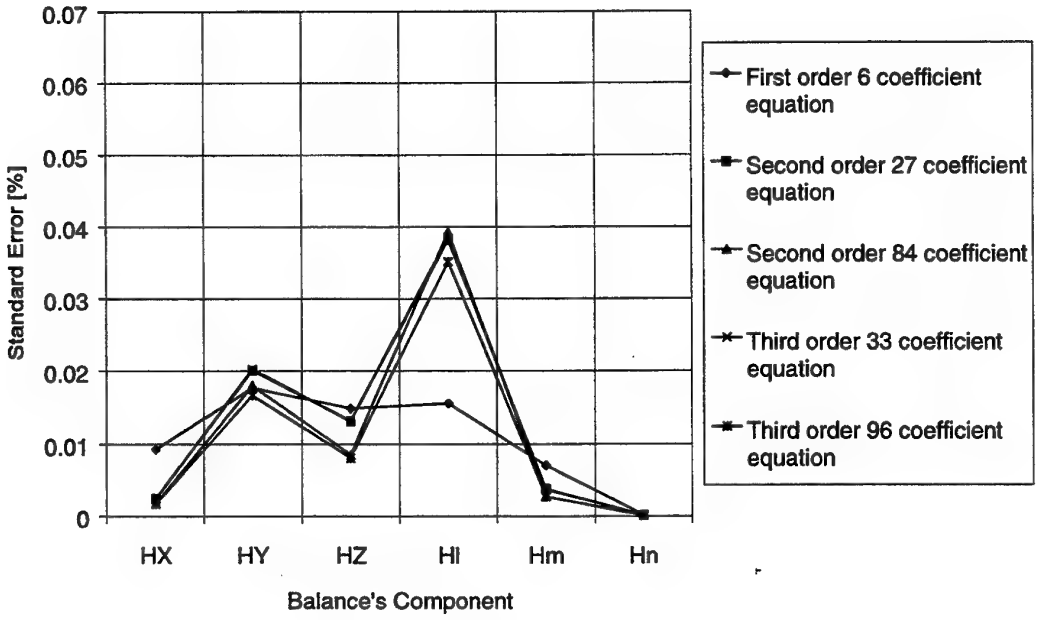


c. 2se Optimisation



d. 3se Optimisation

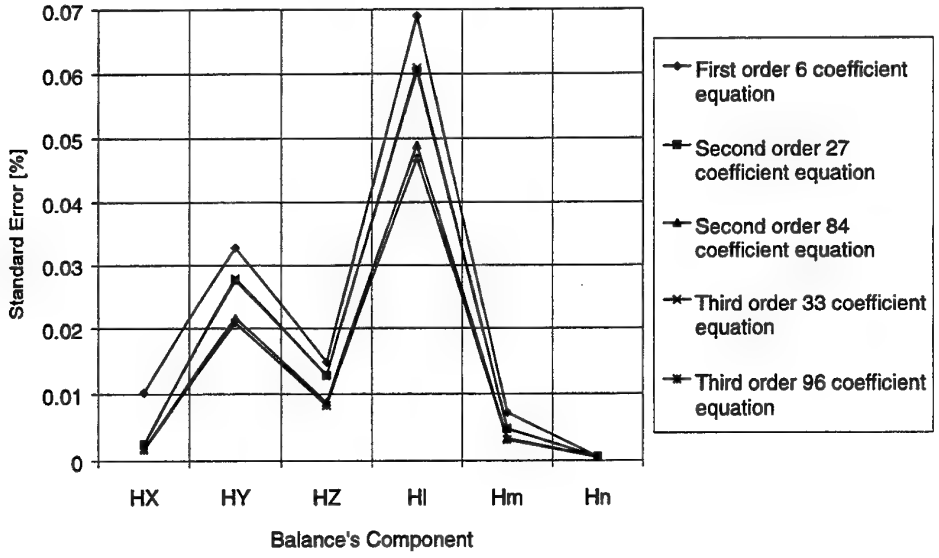
Figure 13. Balance Calibration Model, [R]=[C][H] with 329 data points (cont'd)



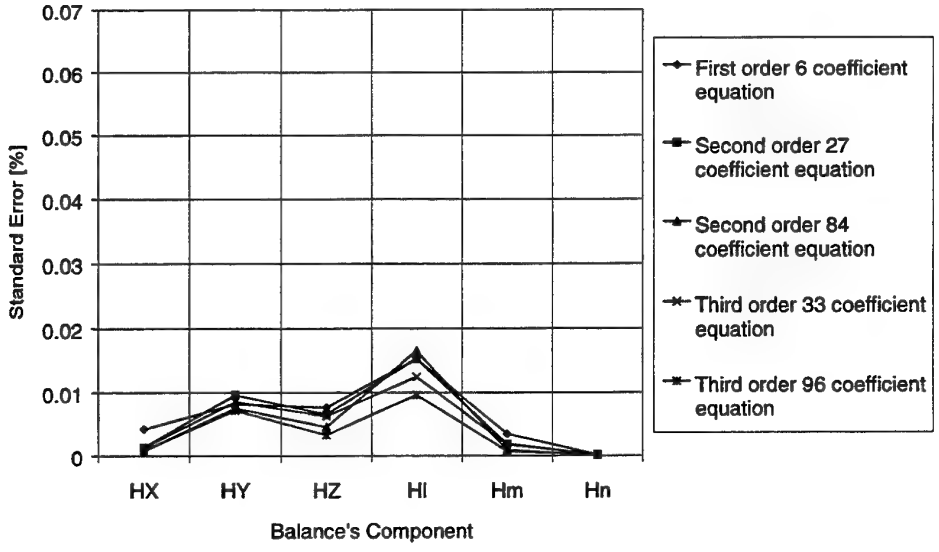
e. Chauvenet's Criterion Optimisation

Figure 13. Balance Calibration Model, [R]=[C][H] with 329 data points (cont'd)

D.5 Calibration Model: [H]=[C][R] (329 calibration data set)

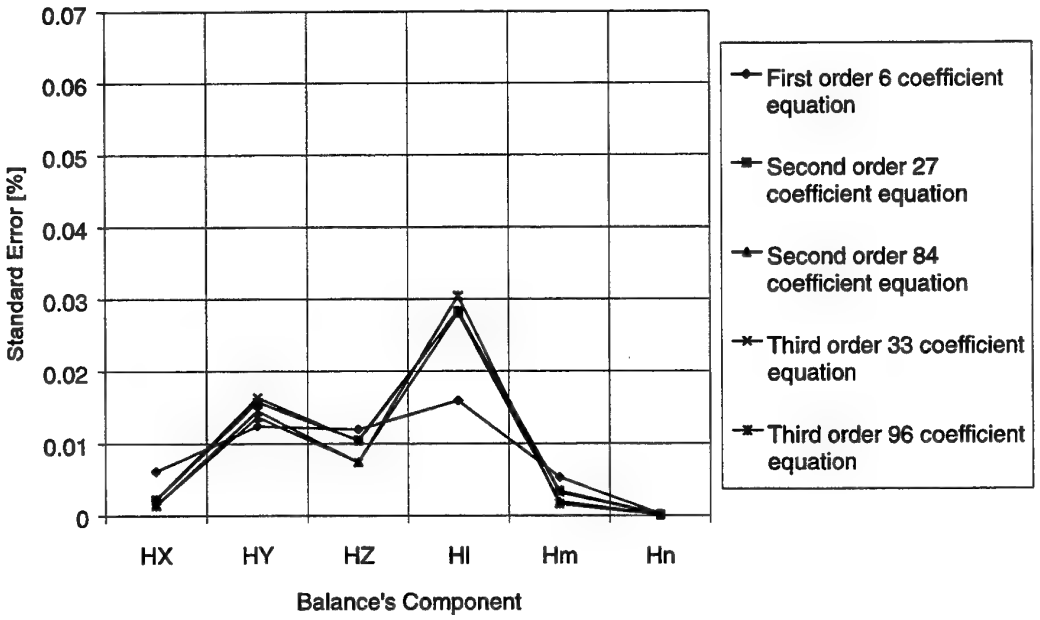


a. No optimisation

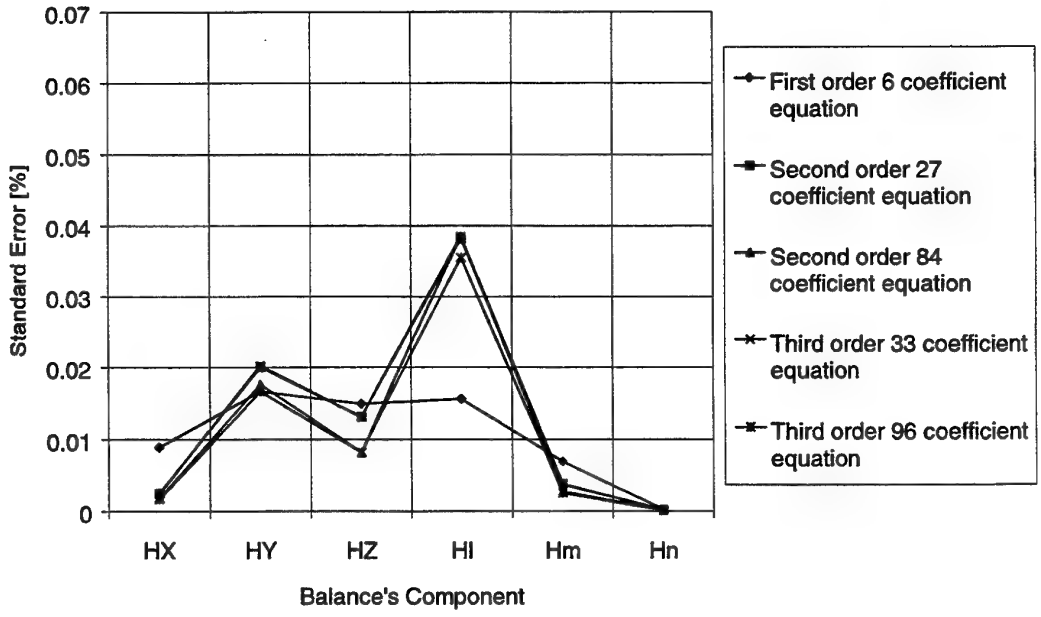


b. 1se Optimisation

Figure 14. Balance Calibration Model: [H]=[C][R] with 329 data points

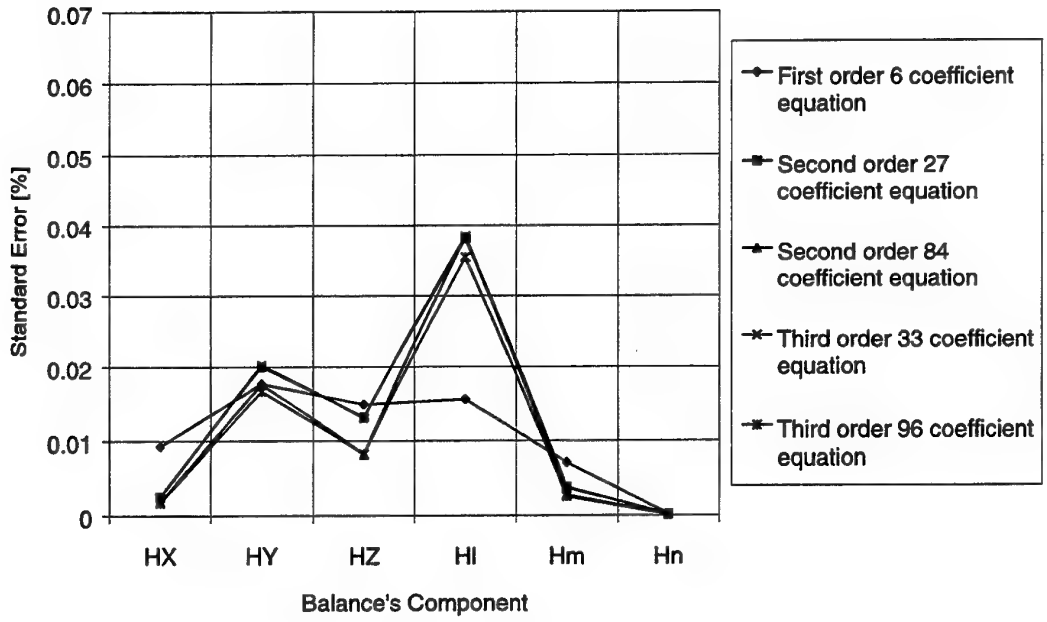


c. 2se Optimisation



d. 3se Optimisation

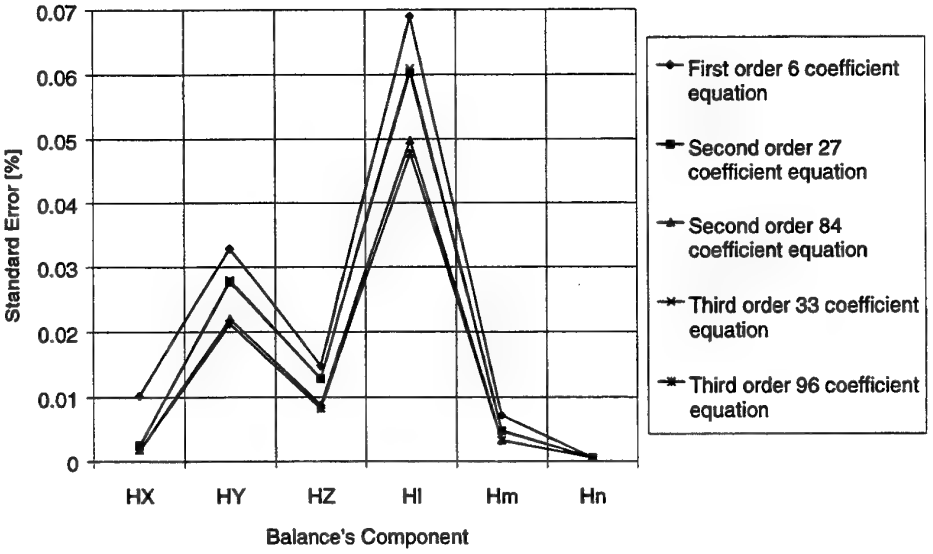
Figure 14. Balance Calibration Model: [H]=[C][R] with 329 data points (cont'd)



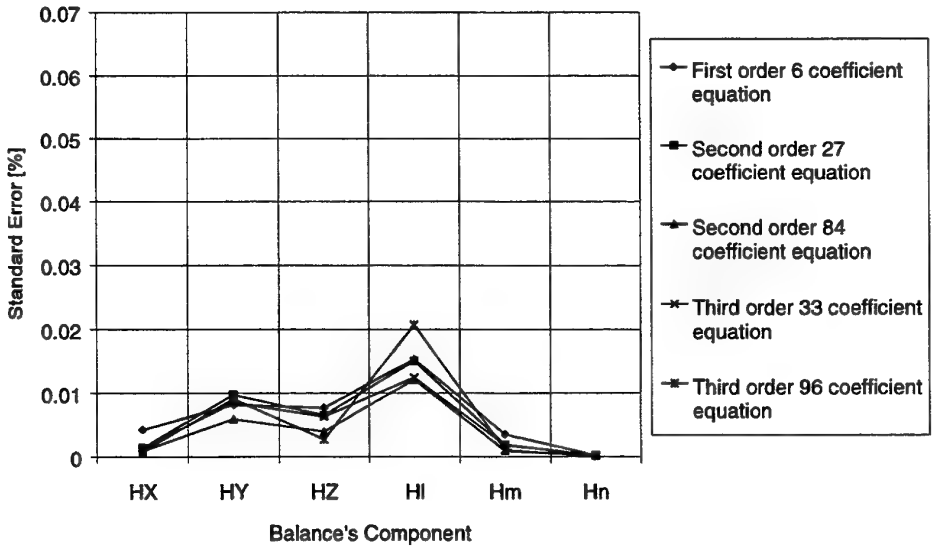
e. Chauvenet's Criterion Optimisation

Figure 14. Balance Calibration Model: [H]=[C][R] with 329 data points (cont'd)

D.6 Calibration Model: [H]=[C][R-H] (329 calibration data set)

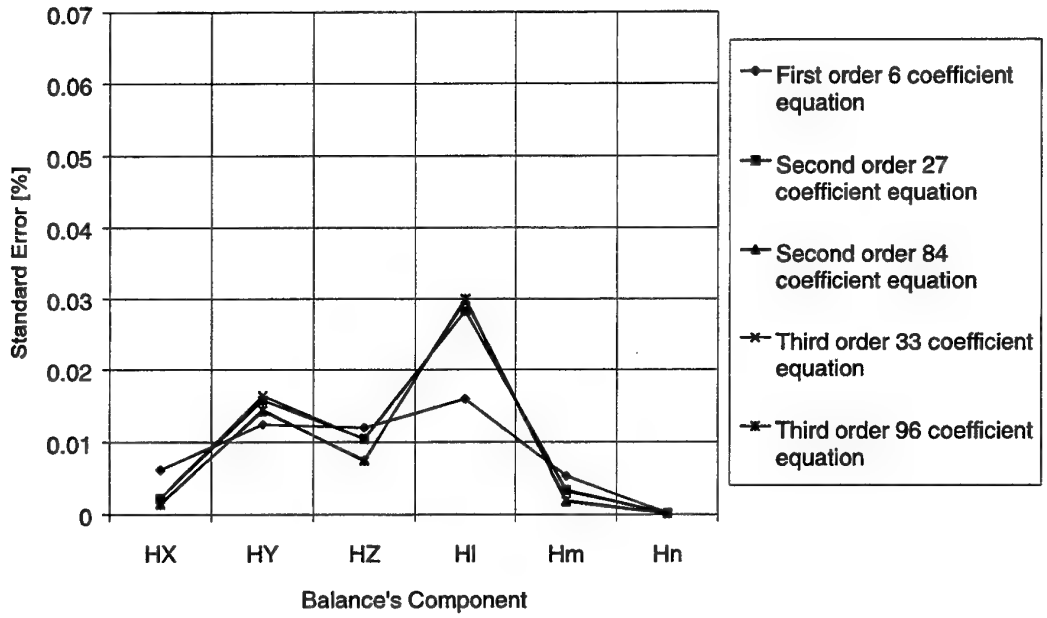


a. No optimisation

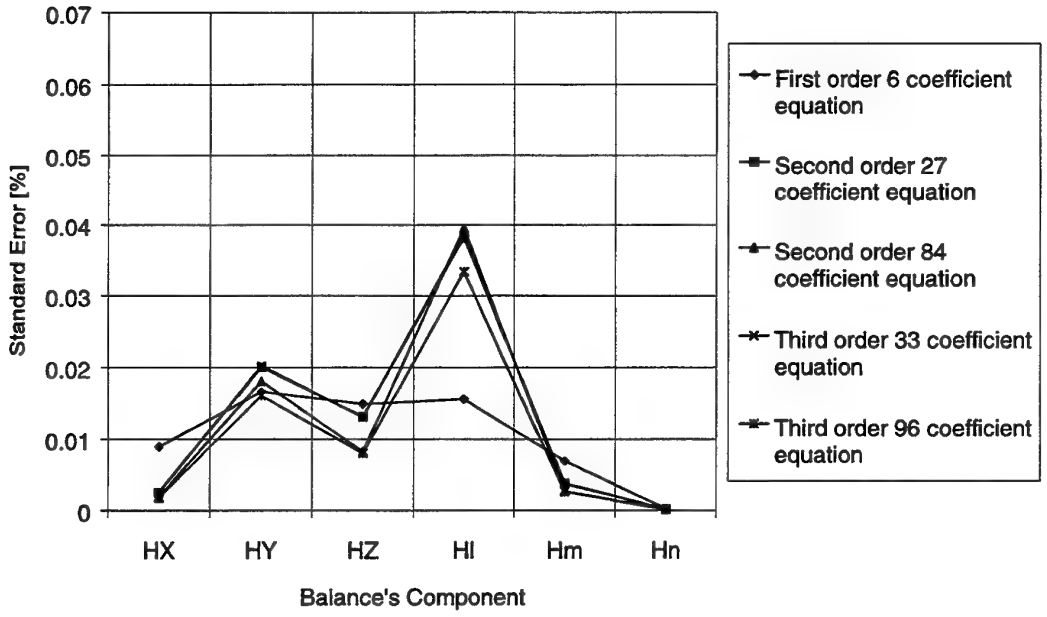


b. 1se Optimisation

Figure 15. Balance Calibration Model: [H]=[C][R-H] with 329 data points

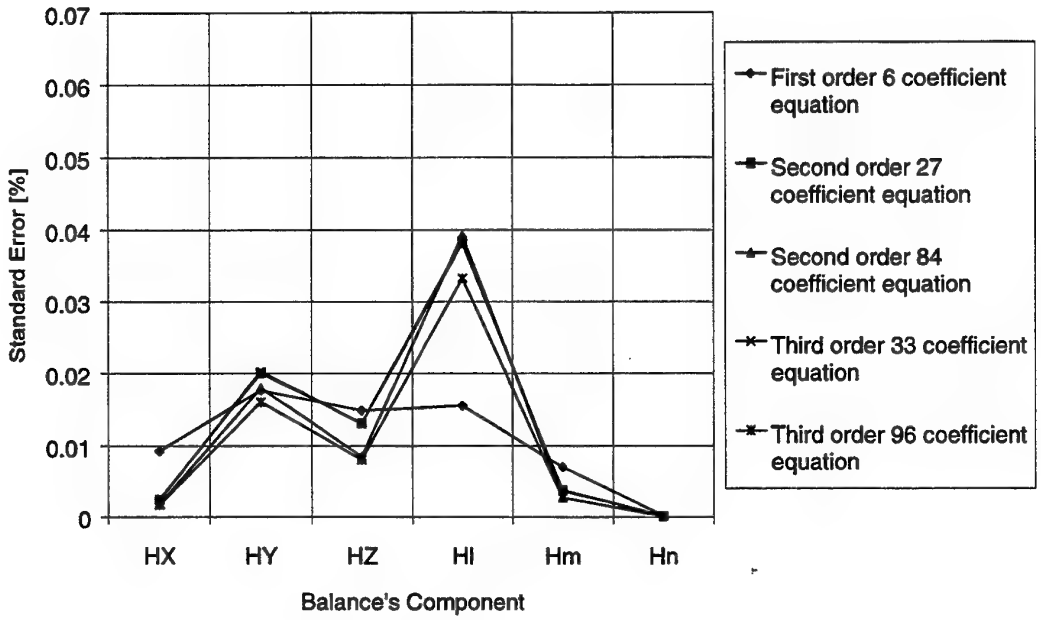


c. 2se Optimisation



d. 3se Optimisation

Figure 15. Balance Calibration Model: [H]=[C][R-H] with 329 data points (cont'd)



e. Chauvenet's Criterion Optimisation

Figure 15. Balance Calibration Model: [H]=[C][R-H] with 329 data points (cont'd)

D.7 5 Component Calibration Model: [R]=[C][H] (1886 calibration data set)

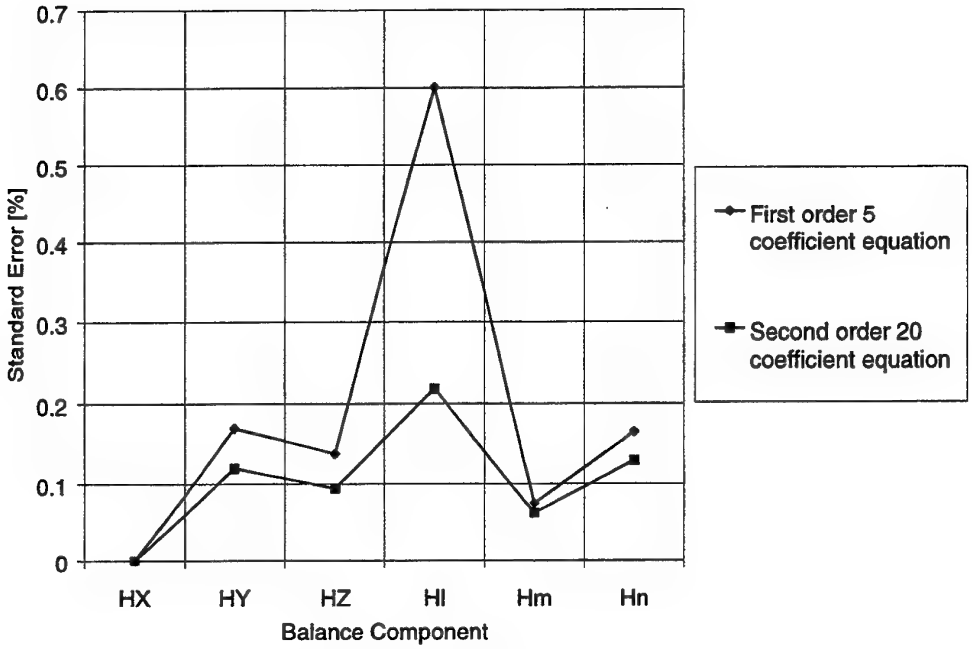


Figure 16. 5 component calibration model: [R]=[C][H] with 1886 data points

D.8 5 Component Calibration Model: [H]=[C][R] (1886 calibration data set)

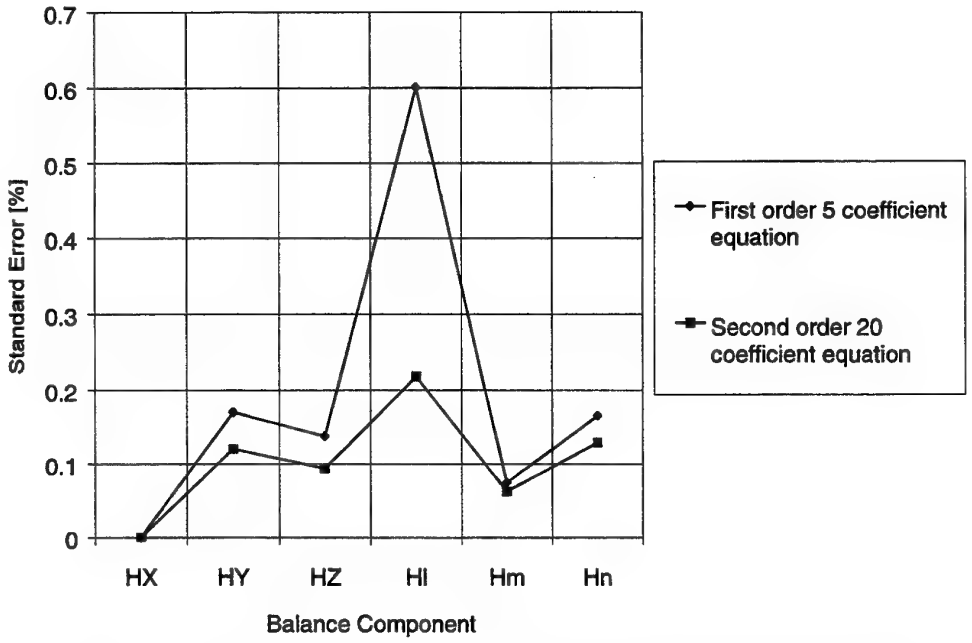


Figure 17. 5 component calibration model: [H]=[C][R] with 1886 data points

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19. ABSTRACT

The construction, comparison and analysis of three distinct strain gauge balance calibration matrix models with various orders of the calibration equations was conducted. The aims of the investigation were to identify the accuracy of the three different calibration matrix models and to analyse their behaviour with different data optimisation techniques. A computer program written in the C and X/Motif programming language has been developed to analyse the matrix models. Two different least squares methods and four optimisation techniques have been implemented within the software. The accuracy of each calibration model is evaluated using two statistical estimation methods. It was found that all three balance calibration models had similar behavior in terms of accuracy. The accuracy of the equation in estimating the loads experienced by the balance increases as the order of the calibration equation increases.